

# M2R Exam

## Fundamentals of data processing and distributed knowledge

### Semantics of Distributed Knowledge part

Duration : 3h

All documents allowed – *No* communication device allowed

January 2023

**Note:** Read all the questions carefully before answering. Justify your answers with respect to the semantics: this is the semantics that justifies their correctness.

Time and points are indicative.

### Course questions

**[Expectation: 30mn; 4pts]**

Here I give only three examples, but it should be around 10 questions, the answers are in the course in general.

Answers to these questions are generally short (if the answer is more than three sentences, it is probably wrong, except for the last one). They are related to the course content.

1. For what is it useful to query different sources?

For accessing/retrieving more information.

2. What does it mean for a structure (formula, theory, network, etc.) to be inconsistent?

It has no model.

3. In modal logic, is  $S \models S'$  defined by  $\forall M, M \models S \Rightarrow M \models S'$  or by  $\forall M, \forall w \in W_M, M, w \models S \Rightarrow M, w \models S'$ ? Does one expression imply the other? Why?

The latter. It entails the former because it will also consider worlds in structures which do not universally satisfy  $S$ .

### Application

**[Expectation: 2h; 14pts]**

#### Triple/graphs

Here is a set of triples (called  $G$ ):

$G$

d:Ringo o2:likes d:Laura  
d:Ringo o2:likes d:Max  
d:Max o2:likes d:Laura  
d:Laura o2:likes d:Max  
d:Laura o2:likes d:Julia

d:Laura o2:hobby d:SurfRidingChamrousse  
d:Laura o2:hobby d:ReadingMadameBovary  
d:Ringo o2:hobby d:DrumPlaying  
d:Max o2:hobby d:HorseRidingCamargue

4. Draw the RDF graphs corresponding to  $G$ .

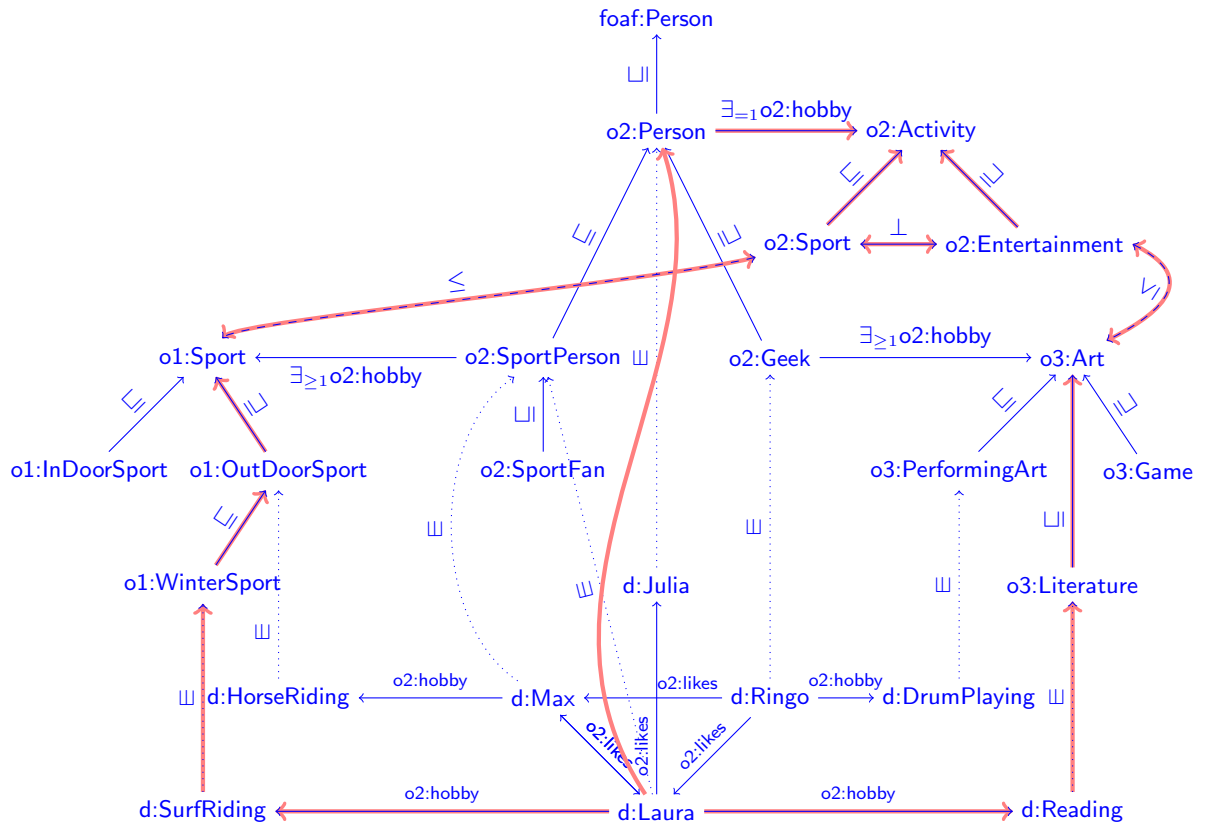
## Ontologies

Consider the three ontologies  $O_1$ ,  $O_2$  and  $O_3$  ( $\sqsubseteq = \text{rdfs:subClassOf}$ ,  $\perp = \text{owl:disjointWith}$ ):

$O_1$	$O_2$
$o1:\text{InDoorSport} \sqsubseteq o1:\text{Sport}$	$o2:\text{Person} \sqsubseteq \text{foaf:Person}$
$o1:\text{OutDoorSport} \sqsubseteq o1:\text{Sport}$	$o2:\text{hobby} \text{ rdfs:domain } o2:\text{Person}$
$o1:\text{WinterSport} \sqsubseteq o1:\text{OutDoorSport}$	$o2:\text{hobby} \text{ rdfs:range } o2:\text{Activity}$
	$o2:\text{Entertainment} \sqsubseteq o2:\text{Activity}$
	$o2:\text{Sport} \sqsubseteq o2:\text{Activity}$
	$o2:\text{Person} \sqsubseteq \exists_{=1} o2:\text{hobby}. o2:\text{Activity}$
	$o2:\text{Sportperson} \sqsubseteq o2:\text{Person} \sqcap \exists_{\geq 1} o2:\text{hobby}. o1:\text{Sport}$
	$o2:\text{Geek} \sqsubseteq o2:\text{Person} \sqcap \exists_{\geq 1} o2:\text{hobby}. o3:\text{Art}$
	$o2:\text{SportFanatic} \sqsubseteq o2:\text{Sportperson} \sqcap \forall o2:\text{likes}. o2:\text{SportPerson}$
$O_3$	
$o3:\text{PerformingArt} \sqsubseteq o3:\text{Art}$	
$o3:\text{Literature} \sqsubseteq o3:\text{Art}$	
$o3:\text{Gaming} \sqsubseteq o3:\text{Art}$	

and its connection to  $G$  ( $\equiv = \text{rdf:type}$ ):

$O_1$	$O_2$	$O_3$
$d:\text{SurfRidingChamrousse} \equiv o1:\text{WinterSport}$	$d:\text{Laura} \equiv o2:\text{Person}$	$d:\text{ReadingMadameBovary} \equiv o3:\text{Literature}$
$d:\text{HorseRidingCamargue} \equiv o1:\text{OutDoorSport}$	$d:\text{Ringo} \equiv o2:\text{Person}$	$d:\text{DrumPlaying} \equiv o3:\text{PerformingArt}$
$d:\text{SailingPaladru} \equiv o1:\text{OutDoorSport}$	$d:\text{Max} \equiv o2:\text{Person}$	
	$d:\text{Julia} \equiv o2:\text{Person}$	



5. Is  $G \cup O_2$  consistent? Either provide a model or discuss the constraints that could prevent one to exist and why they are violated or not.

Yes. It is possible to provide a model in which each individual is interpreted as itself, except `d:SurfRidingChamrousse` and `d:ReadingMadameBovary` interpreted as the same element of  $\Delta$ . Indeed, each of `d:SurfRidingChamrousse`, `d:ReadingMadameBovary`, `d:HorseRidingCamargue` and `d:DrumPlaying` are interpreted as elements of `[o2:Activity]` (because of the `rdfs:range` constraint). Moreover, `d:SurfRidingChamrousse` and `d:ReadingMadameBovary` are both related to `d:Laura` by `o2:hobby`, hence they are interpreted as the same object (due to the  $\exists_{=1}o2:hobby$  constraint on `o2:Person`). There is an additional `o2:Activity` (or one of the existing ones) related to `d:Julia` by `o2:hobby` ( $\exists o \in [o2 : Activity]; \langle d:Julia, o \rangle \in [[o2:hobby]]$ ). This would satisfy all constraints of these ontologies and be a model for RDF or for OWL.

6. Does  $G \cup O_2 \models_{RDF} d:SurfRidingChamrousse \text{ owl:sameAs } d:ReadingMadameBovary$ ?

No. Because there is no `owl:sameAs` statement in  $G \cup O_2$  (see graph above).

7. Does  $G \cup O_2 \models_{OWL} d:SurfRidingChamrousse \text{ owl:sameAs } d:ReadingMadameBovary$ ?

Yes. For the same reason as in Question 5 (`d:Laura` being a `o2:Person`, `o2:Person` having at most one `o2:Activity` as `o2:hobby`, `d:SurfRidingChamrousse` and `d:ReadingMadameBovary` being `o2:Activity` and `d:Laura`'s `o2:hobby`). More semantically,  $G \cup O_2$  contains `d:Laura`  $\sqsubseteq$  `o2:Person` which entails that  $|\{x; \langle d:Laura, o2:hobby, x \rangle\}| = 1$ .  $\langle d:Laura, o2:hobby, d:SurfRidingChamrousse \rangle \in G$  and  $\langle d:Laura, o2:hobby, d:ReadingMadameBovary \rangle \in G$  thus `d:SurfRidingChamrousse` and `d:ReadingMadameBovary` are `o2:Activity` because of the `rdfs:range` constraint. Hence, in any model of  $G \cup O_2$ , `d:SurfRidingChamrousse`' = `d:ReadingMadameBovary`' , thus  $G \cup O_2 \models_{OWL} d:SurfRidingChamrousse \text{ owl:sameAs } d:ReadingMadameBovary$ .

8. Does  $G \cup O_2 \models_{OWL} d:Max \text{ rdf:type } o2:SportFanatic$ ?

No (and this holds for the same reasons for the two next questions). Indeed, nothing prevents to build a model such that `d:HorseRidingCamargue`'  $\in$  `[¬o1:Sport]` (because  $O_1$  is not taken into account). In such a case, `d:Max`'  $\notin$  `[o2:SportPerson]`, and hence `d:Max`'  $\notin$  `[o2:SportFanatic]`.

9. Does  $G \cup O_2 \models_{OWL} d:Ringo \text{ rdf:type } o2:SportFanatic$ ?

No, for the same reason as above with `d:DrumPlaying`.

10. Does  $G \cup O_2 \models_{OWL} d:Laura \text{ rdf:type } \neg o2:SportFanatic$ ?

Neither. For that purpose, it would be necessary that either  $G \cup O_2 \models_{OWL} d:Laura \text{ rdf:type } \neg o2:Sportperson$  or  $G \cup O_2 \models_{OWL} d:Laura \text{ rdf:type } \exists_{\geq 1} o2:likes. \neg o2:Sportperson$ . For the first assertion, it does not hold as nothing prevents that `d:SurfRiding`'  $\in$  `[o1:Sport]`. The second assertion does not hold either because in the same way, nothing prevents that there exists  $o \in [o1:Sport]$  such that  $\langle d:Julia', o \rangle \in [[o2:hobby]]$ . Hence, it is possible to build models not satisfying this assertion.

11. Does  $O_2 \models_{OWL} o2:SportFanatic \sqsubseteq o2:Sportperson$ ?

Yes. `o2:SportFanatic`  $\sqsubseteq$  `o2:Sportperson`  $\sqcap \forall o2:likes. o2:SportPerson \in O_2$ , hence in any model of  $O_2$ , `[o2:SportFanatic]`  $\subseteq$  `[o2:Sportperson]`  $\cap$  `[ $\forall o2:likes. o2:SportPerson$ ]`, thus `[o2:SportFanatic]`  $\subseteq$  `[o2:Sportperson]` which means that  $O_2 \models_{OWL} o2:SportFanatic \sqsubseteq o2:Sportperson$ .

12. Does  $O_2 \models_{OWL} o2:Sportperson \sqsubseteq o2:SportFanatic$ ?

No. For this it would be necessary that all sport person only likes sport persons. Actually, in  $G$  there is the example of `d:Laura` such as for all models of  $G \cup O_2$ , `d:Laura`'  $\in$  `[o2:SportPerson]`,  $\langle d:Laura', d:Julia' \rangle \in [[o2:likes]]$  and `d:Julia`'  $\in$  `[o2:Person]`. The latter statement constrains that there exists a unique  $o \in \Delta$  such that  $\langle d:Julia', o \rangle \in [[o2:hobby]]$ . It is possible to take  $o \in o2:Entertainment$ , i.e.  $o \notin o2:Sport$ . Hence, this model does not satisfy `d:Julia`  $\sqsubseteq$  `o2:SportPerson` because `d:Julia` does not have a `o2:Sport` as `o2:hobby`. That would be a model of  $G \cup O_2$ , thus a model of  $O_2$  in which a `o2:SportPerson` (`d:Laura`) is not a `o2:SportFanatic`.

## Alignments

Consider the following alignments:

$$\begin{array}{ccc}
 A_{12} & & A_{23} \\
 \text{o1:Sport} \leq \text{o2:Sport} & & \text{o2:Entertainment} \geq \text{o3:Art}
 \end{array}$$

13. Does  $A_{23} \models_{\Delta} \text{o2:Activity} \geq \text{o3:PerformingArt}$ ?

Yes. In any model of  $O_2$ ,  $[\text{o2:Entertainment}]_2 \subseteq [\text{o2:Activity}]_2$  and in any model of  $O_3$ ,  $[\text{o3:PerformingArt}]_3 \subseteq [\text{o3:Art}]_3$ . A model of  $A_{23}$  is a pair of models of  $O_2$  and  $O_3$  such that  $[\text{o3:Art}]_3 \subseteq [\text{o2:Entertainment}]_2$ . Hence, in any model of  $A_{23}$ ,  $[\text{o3:PerformingArt}]_3 \subseteq [\text{o2:Activity}]_2$ , and thus  $A_{23} \models_{\Delta} \text{o2:Activity} \geq \text{o3:PerformingArt}$ .

14. Does  $A_{12} \models_{\Delta} \text{o2:Sportperson} \sqsubseteq \text{o2:Person} \sqcap \exists_{\geq 1} \text{o2:hobby.o2:Sport}$ ?

No.  $A_{12}$  selects pairs of models in which  $[\text{o2:Sport}]_2 \supseteq [\text{o1:Sport}]_1$ . Elements of  $\text{o2:Sportperson}$  must have at least one  $\text{o2:hobby}$  which is a  $\text{o1:Sport}$ . But there may be  $\text{o2:Sport}$  which are not  $\text{o1:Sport}$ . Hence it is possible to satisfy  $[\text{o2:Person}]_2 \cap \{o; |\{h \in [\text{o2:Sport}]_2; \langle o, h \rangle \in [[\text{o2:hobby}]]_2\}| \geq 1\}$  but not  $[\text{o2:Person}]_2 \cap \{o; |\{h \in [\text{o1:Sport}]_1; \langle o, h \rangle \in [[\text{o2:hobby}]]_2\}| \geq 1\}$ .

## Belief revision

Consider that we add:

$$\text{o2:Sport owl:disjointWith o2:Entertainment.}$$

to  $O_2$ .

15. Does this make  $G \cup O_2$  inconsistent? Why?

No. This could be the case if one individual would necessarily (in all models) belong to both  $\text{o2:Sport}$  and  $\text{o2:Entertainment}$ . The only such candidates in  $G$  would be the four persons, but nothing constrain them to be activities, and the four hobbies. However, none of these activities are entailed by  $G \cup O_2$  to belong to any of  $\text{o2:Sport}$  or  $\text{o2:Entertainment}$ .

16. Does  $\langle \{O_1, O_2 \cup G, O_3\} \{A_{12}, A_{23}\} \rangle \models_{\Delta} \text{o1:Sport} \perp \text{o3:Art}$ ?

Yes. The models of the network (which are triples of models of  $O_1$ ,  $G \cup O_2$  and  $O_3$  respectively) are individual models of the ontologies satisfying the alignments. Hence, they satisfy  $[\text{o1:Sport}]_1 \subseteq [\text{o2:Sport}]_2$  and  $[\text{o2:Entertainment}]_2 \supseteq [\text{o3:Art}]_3$  but they also satisfy  $[\text{o2:Sport}]_2 \cap [\text{o2:Entertainment}]_2 = \emptyset$ . Thus, they all satisfy  $[\text{o1:Sport}]_1 \cap [\text{o3:Art}]_3 = \emptyset$  which means that they entail  $\text{o1:Sport} \perp \text{o3:Art}$ .

17. Does this make  $\langle \{O_1, O_2 \cup G, O_3\} \{A_{12}, A_{23}\} \rangle$  inconsistent? Why?

Yes. Because in all models of the network we have:  $d:\text{SurfRidingChamrousse}' \in [\text{o1:Sport}]_1$  hence  $d:\text{SurfRidingChamrousse}' \in [\text{o2:Sport}]_2$  (for satisfying  $A_{12}$ ) and  $d:\text{ReadingMadameBovary}' \in [\text{o3:Art}]_3$  and thus  $d:\text{ReadingMadameBovary}' \in [\text{o2:Entertainment}]_2$  for satisfying  $A_{23}$ . But as of Question 7,  $G \cup O_2 \models_{OWL} d:\text{SurfRidingChamrousse} \text{ owl:sameAs } d:\text{ReadingMadameBovary}$ , thus for all models of this network  $d:\text{ReadingMadameBovary}' \in [\text{o2:Sport}]_2 \cap [\text{o2:Entertainment}]_2$  and  $[\text{o2:Sport}]_2 \cap [\text{o2:Entertainment}]_2 = \emptyset$ . So the network has no model.

18. What are the statements that can be suppressed to restore consistency?

These are all the 14 statements in red in the diagram above.

## Epistemic logic

19. Model the ontologies (without the last axiom added to  $O_2$ ) and alignments in epistemic logic as in Section 7.6 of the course (Modelling the alignment repair game). This means that three agents are considered each one having an ontology.

Here are the knowledge axioms provided by  $\tau$ :

$K_1$ o1:InDoorSport $\sqsubseteq$ o1:Sport	$K_1$ o1:OutDoorSport $\sqsubseteq$ o1:Sport
$K_1$ o1:WinterSport $\sqsubseteq$ o1:OutDoorSport	$K_1$ o1:WinterSport(d:SurfRidingChamrousse)
$K_1$ o1:OutDoorSport(d:HorseRidingCamargue)	$K_1$ o1:OutDoorSport(d:SailingPaladru)
$K_3$ o3:Literature(d:ReadingMadameBovary)	$K_3$ o3:PerformingArt(d:DrumPlaying)
$K_3$ o3:PerformingArt $\sqsubseteq$ o3:Art	$K_3$ o3:Literature $\sqsubseteq$ o3:Art
$K_3$ o3:Gaming $\sqsubseteq$ o3:Art	$K_2$ o2:Person $\sqsubseteq$ foaf:Person
$K_2$ o2:hobby rdfs:domain o2:Person	$K_2$ o2:hobby rdfs:range o2:Activity
$K_2$ o2:Entertainment $\sqsubseteq$ o2:Activity	$K_2$ o2:Sport $\sqsubseteq$ o2:Activity
$K_2$ o2:Person(d:Laura)	$K_2$ o2:Person(d:Ringo)
$K_2$ o2:Person(d:Max)	$K_2$ o2:Person(d:Julia)

these axioms were not specified in the course and no semantics was given to them:

$K_2$ o2:Person $\sqsubseteq$ $\exists_{=1}$ o2:hobby.o2:Activity	
$K_2$ o2:Geek $\sqsubseteq$ o2:Person $\sqcap$ $\exists_{\geq 1}$ o2:hobby.o3:Art	
$K_2$ o2:Sportperson $\sqsubseteq$ o2:Person $\sqcap$ $\exists_{\geq 1}$ o2:hobby.o1:Sport	
$K_2$ o2:SportFanatic $\sqsubseteq$ o2:Sportperson $\sqcap$ $\forall$ o2:likes.o2:SportPerson	
$K_2$ o2:likes(d:Ringo, d:Laura)	$K_2$ o2:likes(d:Ringo, d:Max)
$K_2$ o2:likes(d:Max, d:Laura)	$K_2$ o2:likes(d:Laura, d:Max)
$K_2$ o2:likes(d:Laura, d:Julia)	$K_2$ o2:hobby(d:Laura, d:SurfRidingChamrousse)
$K_2$ o2:hobby(d:Laura, d:ReadingMadameBovary)	$K_2$ o2:hobby(d:Ringo, d:DrumPlaying)
$K_2$ o2:hobby(d:Max, d:HorseRidingCamargue)	

and for the beliefs:

$B_1$ o1:Sport $\sqsubseteq$ o2:Sport	$B_2$ o1:Sport $\sqsubseteq$ o2:Sport
$B_3$ o2:Entertainment $\sqsubseteq$ o3:Art	$B_2$ o2:Entertainment $\sqsubseteq$ o3:Art

20. What would the effect of the announcement of  $o2:Sport$  owl:disjointWith  $o2:Entertainment$  be?

The result of the announcement is that each agent suppress from its models all worlds in which the intersection between the interpretation of the two classes is not empty. Agents 1 and 3 do not have constraints on  $o2:Activity$ , hence this should not change what they entail except the fact that the announced formula is now knowledge for them. In what concerns Agent 2, in all its models it holds that:

$o2:Person \sqsubseteq \exists_{=1}o2:hobby.o2:Activity$   
 $o2:Person(d:Laura)$   
 $o2:Entertainment \sqsubseteq o2:Activity$   
 $o2:Sport \sqsubseteq o2:Activity$   
 $o2:Sport$  owl:disjointWith  $o2:Entertainment$   
 $o2:hobby(d:Laura, d:SurfRidingChamrousse)$   
 $o2:hobby(d:Laura, d:ReadingMadameBovary)$

Moreover, in the most plausible worlds of its information cell holds:

$o1:Sport \sqsubseteq o2:Sport$   
 $o2:Entertainment \sqsubseteq o3:Art$

However, nothing constrain that (only known by 3):

$o3:Literature(d:ReadingMadameBovary)$   
 $o3:PerformingArt \sqsubseteq o3:Art$   
 $o3:Literature \sqsubseteq o3:Art$

Nor that (only known by 1):

$o1:WinterSport(d:SurfRidingChamrousse)$   
 $o1:OutDoorSport \sqsubseteq o1:Sport$   
 $o1:WinterSport \sqsubseteq o1:OutDoorSport$

The announcement will destroy all the worlds in which all this information holds together because it is inconsistent with it. There will however remain worlds in which this is true

## Open question

[Expectation: 20mn; 3pts]

A type of belief revision is partial meet revision which computes the intersection between selected maximal consistent subtheories. One problem is to define how to select these subtheories. Cultural knowledge evolution applies a simple adaptation operator (similar to selecting one theory) to restore local consistency. Could you imagine how to use the cultural knowledge evolution approach to ‘perform’ partial meet revision?

One can consider that each agent choose at random one maximal consistent sub-theory. This would make a maxichoice revision. Then, following the cultural evolution, some of these theories will have to be changed again because they are not good enough. This means that some of them would be abandoned. The remaining ones would be those theories consider by the set of agents. In this sense, natural selection, or rather cultural selection, would play the rôle of the selection operation of partial meet revision.

Of course, since agents would not be forced to adopt the intersection of the theories, this is not *stricto sensu* partial meet revision.