

# Corrected and annotated M2R Exam

## Fundamentals of data processing and distributed knowledge

### Semantics of Distributed Knowledge part

Duration : 2h15

All documents allowed – *No* communication device allowed

January 2022

**Note:** Read all the questions carefully before answering. Do not hesitate to justify your answers.  
Time and points are indicative.

### Course questions

**[Expectation: 30mn; 5pts]**

Here I give only three examples, but it should be around 10 questions, the answers are in the course in general.

Answers to these questions are generally short (if the answer is more than three sentences, it is probably wrong, except for the last one). They are related to the course content.

1. For what is it useful to query different sources?

[For accessing/retrieving more information.](#)

2. What does it mean for a structure (formula, theory, network, etc.) to be inconsistent?

[It has no model.](#)

3. In modal logic, is  $S \models S'$  defined by  $\forall M, M \models S \Rightarrow M \models S'$  or by  $\forall M, \forall w \in W_M, M, w \models S \Rightarrow M, w \models S'$ ? Does one expression imply the other? Why?

[The latter. It entails the former because it will also consider worlds in structures which do not universally satisfy  \$S\$ .](#)

### Application

**[Expectation: 1h45; 15pts]**

We will consider three simple sources about sports.

### Triple/graphs

Here are three sets of triples:

$G_2$

d:football rdf:type o2:outdry  
d:handball rdf:type o2:indry  
d:natation rdf:type o2:inwet

$G_1$

d:football o1:influences d:handball  
d:handball o1:influences d:waterpolo  
d:natation o1:influences d:waterpolo

$G_3$

d:football rdf:type o3:grdcoll  
d:natation rdf:type o3:wtrind  
d:waterpolo rdf:type o3:wtrcoll

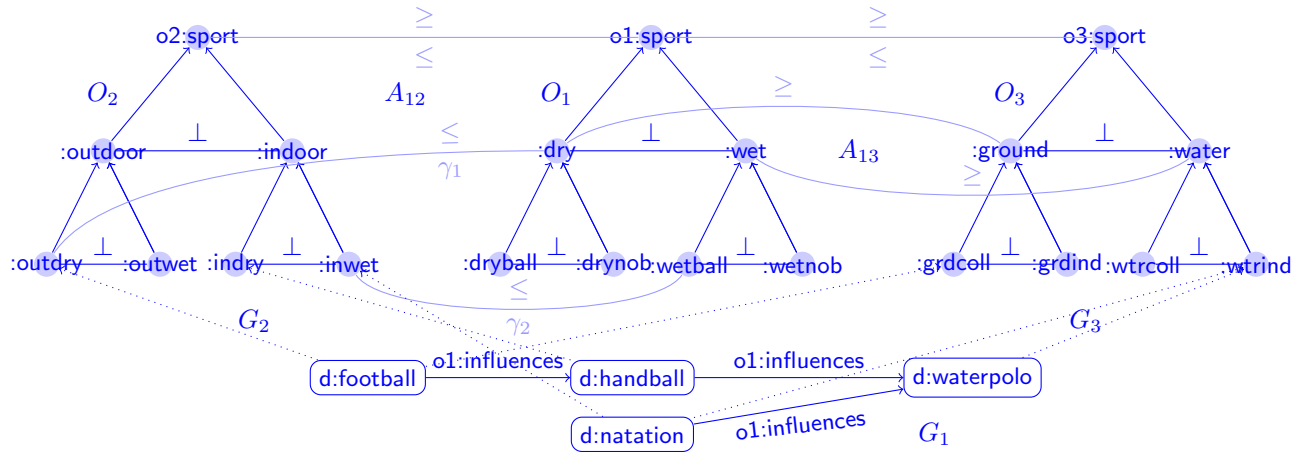


Figure 1: The initial situation.

4. Draw the RDF graphs corresponding to  $G_1$  and  $G_3$ .

See Figure 1 (bottom for the assertions of  $G_1$  and the `rdf:type` links from resources of the bottom to classes for  $G_2$  and  $G_3$ ).

5. Does  $G_1 \models_{RDF} \text{d:waterpolo } \text{rdf:type } \text{o1:wet}$ ?

No. There is no assertion involving `o1:wet` in  $G_1$ .

6. Does  $G_1 \cup G_3 \models_{RDF} \text{d:waterpolo } \text{rdf:type } \text{o3:wtrcoll}, \text{d:handball } \text{o1:influences } \text{d:waterpolo}$ ?

Yes. These are actually triples of these graphs.

## Ontologies

Each source has a different ontology based on different criteria. They consider sports as either individual or collective, wet or dry, indoor or outdoor, ball-based or not, etc.

Consider the ontologies  $O_1$ ,  $O_2$  and  $O_3$  ( $\sqsubseteq = \text{rdfs:subClassOf}$ ,  $\perp = \text{owl:disjointWith}$ ):

| $O_2$  | $O_1$   | $O_3$  |
|--|---|--|
| <code>o2:outdoor</code> $\sqsubseteq$ <code>o2:sport</code>  | <code>o1:dry</code> $\sqsubseteq$ <code>o1:sport</code>   | <code>o3:ground</code> $\sqsubseteq$ <code>o3:sport</code>   |
| <code>o2:indoor</code> $\sqsubseteq$ <code>o2:sport</code>   | <code>o1:wet</code> $\sqsubseteq$ <code>o1:sport</code>   | <code>o3:water</code> $\sqsubseteq$ <code>o3:sport</code>    |
| <code>o2:outdoor</code> $\perp$ <code>o2:indoor</code>       | <code>o1:dry</code> $\perp$ <code>o1:wet</code>           | <code>o3:ground</code> $\perp$ <code>o3:water</code>         |
| <code>o2:outdry</code> $\sqsubseteq$ <code>o2:outdoor</code> | <code>o1:dryball</code> $\sqsubseteq$ <code>o1:dry</code> | <code>o3:grdcoll</code> $\sqsubseteq$ <code>o3:ground</code> |
| <code>o2:outwet</code> $\sqsubseteq$ <code>o2:outdoor</code> | <code>o1:drynob</code> $\sqsubseteq$ <code>o1:dry</code>  | <code>o3:grdind</code> $\sqsubseteq$ <code>o3:ground</code>  |
| <code>o2:outdry</code> $\perp$ <code>o2:outwet</code>        | <code>o1:dryball</code> $\perp$ <code>o1:drynob</code>    | <code>o3:grdcoll</code> $\perp$ <code>o3:grdind</code>       |
| <code>o2:indry</code> $\sqsubseteq$ <code>o2:indoor</code>   | <code>o1:wetball</code> $\sqsubseteq$ <code>o1:wet</code> | <code>o3:wtrcoll</code> $\sqsubseteq$ <code>o3:water</code>  |
| <code>o2:inwet</code> $\sqsubseteq$ <code>o2:indoor</code>   | <code>o1:wetnob</code> $\sqsubseteq$ <code>o1:wet</code>  | <code>o3:wtrind</code> $\sqsubseteq$ <code>o3:water</code>   |
| <code>o2:indry</code> $\perp$ <code>o2:inwet</code>          | <code>o1:wetball</code> $\perp$ <code>o1:wetnob</code>    | <code>o3:wtrcoll</code> $\perp$ <code>o3:wtrind</code>       |

7. Draw the ontologies as three RDF graphs representing their hierarchy.

See Figure 1.

8. Does  $O_1 \models_{OWL} \text{o1:wetball} \perp \text{o1:dry}$ ?

Yes. Because  $\text{o1:wetball} \sqsubseteq \text{o1:wet} \perp \text{o1:dry}$ . Or, semantically, because for any model  $m$  of  $O_1$ ,  $m(\text{o1:wetball}) \subseteq m(\text{o1:wet})$  and  $m(\text{o1:wet}) \cap m(\text{o1:dry}) = \emptyset$ , hence  $m(\text{o1:wetball}) \cap m(\text{o1:dry}) = \emptyset$  and since this holds for every model of  $O_1$ , then  $O_1 \models_{OWL} \text{o1:wetball} \perp \text{o1:dry}$ .

9. Does  $G_1 \cup O_1 \models_{OWL} \text{d:waterpolo} \text{ rdf:type } \text{o1:wet}$ ?

No. Because there is no assertion concerning  $\text{d:waterpolo}$ , which could entail a type assertion (e.g.  $\langle \text{o1:influences, rdfs:range, o1:wet} \rangle$ ).

10. Does  $G_1 \cup G_3 \cup O_1 \cup O_3 \models_{OWL} \text{d:waterpolo} \text{ rdf:type } \text{o3:water. d:handball o1:influences d:waterpolo}$ ?

Yes.  $\text{d:waterpolo} \text{ rdf:type } \text{o3:wtrcol} \sqsubseteq \text{o3:water}$  (from  $G_3 \cup O_3$ ) and  $\text{d:handball o1:influences d:waterpolo}$  (from  $G_1$ ).

## Query

Some agents would like to take advantage of these three sources and answer queries. Consider the three following queries:

| $q_1$   | $q_2$  | $q_3$  |
|---|--|--|
| SELECT $?x, ?y$<br>WHERE $?x \text{ o1:influences } ?y$ | SELECT $?x, ?y$<br>WHERE<br>$?x \text{ o1:influences } ?y$<br>$?y \text{ rdf:type } \text{o3:water}$ | SELECT $?x, ?y$<br>WHERE<br>$?x \text{ o1:influences } ?y$<br>$?x \text{ rdf:type } \text{o1:dry}$<br>$?y \text{ rdf:type } \text{o1:wet}$ |

11. What are the results of evaluating  $q_1$ ,  $q_2$  and  $q_3$  against  $O_1 \cup G_1$  ( $\mathcal{A}(q[?x, ?y], O_1 \cup G_1)$ )?

| $q_1$   | $q_2$ | $q_3$ |
|---|-------|-------|
| $\langle \text{d:football, d:handball} \rangle$<br>$\langle \text{d:handball, d:waterpolo} \rangle$<br>$\langle \text{d:natation, d:waterpolo} \rangle$ |       |       |

12. What are the results of evaluating  $q_1$ ,  $q_2$  and  $q_3$  against  $O_1 \cup O_2 \cup O_3 \cup G_1 \cup G_2 \cup G_3$  ( $\mathcal{A}^{O_1 \cup O_2 \cup O_3}(q[?x, ?y], G_1 \cup G_2 \cup G_3)$ )?

| $q_1$   | $q_2$  | $q_3$ |
|---|--|-------|
| $\langle \text{d:football, d:handball} \rangle$<br>$\langle \text{d:handball, d:waterpolo} \rangle$<br>$\langle \text{d:natation, d:waterpolo} \rangle$ | $\langle \text{d:handball, d:waterpolo} \rangle$<br>$\langle \text{d:natation, d:waterpolo} \rangle$ |       |

## Alignments

Consider the following alignments between  $O_1$  and the two other ontologies:

$A_{12} = \{ \text{o1:sport} \geq \text{o2:sport}, \text{o1:sport} \leq \text{o2:sport}, \text{o1:dry} \geq \text{o2:outdry}, \text{o1:wetball} \geq \text{o2:inwet} \}$

$A_{13} = \{ \text{o1:sport} \geq \text{o3:sport}, \text{o1:sport} \leq \text{o3:sport}, \text{o1:dry} \geq \text{o3:ground}, \text{o1:wet} \geq \text{o3:water} \}$

13. Does  $A_{13} \models_{\Delta} \langle \text{o1:wet, } \geq, \text{o3:wtrcoll} \rangle$ ?

Yes. Because for any pair of models  $\langle m_1, m_3 \rangle$  of  $\langle O_1, O_3 \rangle$ ,  $A_{13}$  constrains that  $m_1(\text{o1:wet}) \supseteq m_3(\text{o3:water})$  and for any model  $m_3$  of  $O_3$ ,  $m_3(\text{o3:wtrcoll}) \subseteq m_3(\text{o3:water})$ .

14. Does  $\langle \{O_1 \cup G_1, O_3 \cup G_3\}, \{A_{13}\} \rangle \models_{\Delta} \text{d:waterpolo} \text{ rdf:type } \text{o1:wet}$  (or  $\langle \text{d:waterpolo, rdf:type, o1:wet} \rangle \in \text{Cn}_{\langle \{O_1 \cup G_1, O_3 \cup G_3\}, \{A_{13}\} \rangle}^{\omega}$ )?

Yes. Because for any model  $m_3$  of  $O_3$ ,  $m_3(\text{d:waterpolo}) \in m_3(\text{o3:wtrcoll})$  and the answer above means that  $m_1(\text{d:waterpolo}) \in m_1(\text{o1:wet})$ .

## Alignment creation game

In the alignment creation game, agents know the leaf class names of other agents' ontologies, but not the definition of these classes, nor subclass (subsumption). The game is played in the following way:

- an agent  $a$  asks another agent  $b$  to show some object of class  $c_b$ ;
- agent  $b$  answers with an object  $o$ ;
- agent  $a$  identifies in its own ontology the most specific class  $c'_a$  to which  $o$  belongs;
- if  $A_{ab}$  does not entail any correspondence  $\langle c_a, \geq, c_b \rangle$ , then the outcome of the game is UNKNOWN and agent  $a$  adds  $\langle c'_a, \geq, c_b \rangle$  to  $A_{ab}$ ;
- if  $\langle c_a, \geq, c_b \rangle \in A_{ab}$  then
  - If  $c'_a \sqsubseteq c_a$ , then the game is a SUCCESS;
  - Otherwise, the game is a FAILURE and agent  $a$  replaces  $\langle c'_a, \geq, c_b \rangle$  in  $A_{ab}$  by  $\langle c''_a, \geq, c_b \rangle$  with  $c''_a$  the most specific superclass of  $c_a$  and  $c'_a$ .

Consider  $A_{12}$  between  $O_1$  and  $O_2$  and that agent 1 has received from agent 2 the following set of objects:

| $c_2$     | Object      | $c'_1$     | status | action |
|-----------|-------------|------------|--------|--------|
| o2:outdry | d:marathon  | o1:drynob  |        |        |
| o2:outwet | d:triathlon | o1:drynob  |        |        |
| o2:indry  | d:basket    | o1:dryball |        |        |
| o2:inwet  | d:diving    | o1:drynob  |        |        |
| o2:inwet  | d:waterpolo | o1:wetball |        |        |
| o2:indry  | d:judo      | o1:drynob  |        |        |

15. Trace the game by showing how it evolves the alignment  $A_{12}$ . I.e. fill the table above (status=SUCCESS, FAILURE, UNKNOWN).

| $c_2$     | Object      | $c'_1$     | status                 | action   |
|-----------|-------------|------------|------------------------|--|
| o2:outdry | d:marathon  | o1:drynob  | SUCCESS ( $\gamma_1$ ) | none   |
| o2:outwet | d:triathlon | o1:drynob  | UNKNOWN                | add $\gamma_3 = \langle o1:drynob, \geq, o2:outwet \rangle$            |
| o2:indry  | d:basket    | o1:dryball | UNKNOWN                | add $\gamma_4 = \langle o1:dryball, \geq, o2:indry \rangle$            |
| o2:inwet  | d:diving    | o1:drynob  | FAILURE ( $\gamma_2$ ) | repl $\gamma_2$ by $\gamma_5 = \langle o1:wet, \geq, o2:inwet \rangle$ |
| o2:inwet  | d:waterpolo | o1:wetball | SUCCESS ( $\gamma_5$ ) | none   |
| o2:indry  | d:judo      | o1:drynob  | FAILURE ( $\gamma_4$ ) | repl $\gamma_4$ by $\gamma_6 = \langle o1:dry, \geq, o2:indry \rangle$ |

16. What is the content of  $A'_{12}$ , the evolution of  $A_{12}$  after these 6 games?

$$A'_{12} = \{o1:sport \geq o2:sport, o1:sport \leq o2:sport, o1:dry \geq o2:outdry, \\ o1:wet \geq o2:inwet, o1:drynob \geq o2:outwet, o1:dry \geq o2:indry\}$$

This is also displayed in Figure 2.

17. Is the proposed adaptation always correct?

Yes. Because the discarded correspondence is always incorrect. But the added correspondences may be too specific, as shown in the example of  $\gamma_4$  above. They may be invalidated by a later game.

18. Does it converge to a stable alignment?

Once all possible object types have been played, all remaining correspondences should not create any more failures. However, in this game we have no way to know if they all have been explored (contrary to ARG).

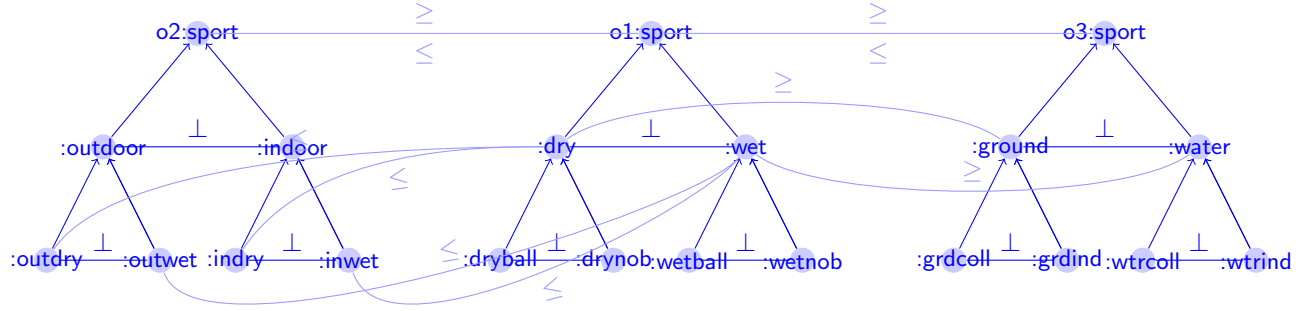


Figure 2: The corrected situation... with  $A'_{12}$ .

## Query evaluation revisited

19. Provide a decomposition of query  $q_3$  that an agent using ontology  $O_1$  could use in order to evaluate it with respect the alignments  $A'_{12}$  and  $A_{13}$ ?

$q_3$ , may be partitioned into three queries  $q_3^1 = \{ \langle ?x, o1:influences, ?y \rangle \}$ ,  $q_3^2 = \{ \langle ?x, rdf:type, o2:indry \rangle \}$  and  $q_3^3 = \{ \langle ?y, rdf:type, o3:water \rangle \}$ .

20. Provide the answer of each subquery and how it provides an answer to  $q_3$ .

- $\mathcal{A}^{O_1}(q_3^1(\langle ?x, ?y \rangle), G_1) = \mathcal{A}^{O_1}(q_1(\langle ?x, ?y \rangle), G_1)$ ,  
i.e.  $\{ \langle d:football, d:handball \rangle, \langle d:handball, d:waterpolo \rangle, \langle d:natation, d:waterpolo \rangle \}$ ,
- $\mathcal{A}^{O_2}(q_3^2(\langle ?x \rangle), G_2) = \{ \langle d:handball \rangle \}$
- $\mathcal{A}^{O_3}(q_3^3(\langle ?y \rangle), G_3) = \{ \langle d:waterpolo \rangle \}$

Hence,  $\mathcal{A}^{O_1}(q_3^1(\langle ?x, ?y \rangle), G_1) \bowtie \mathcal{A}^{O_2}(q_3^2(\langle ?x \rangle), G_2) \bowtie \mathcal{A}^{O_3}(q_3^3(\langle ?y \rangle), G_3) = \{ \langle d:handball, d:waterpolo \rangle \}$ .

## Epistemic logic model of agent knowledge

21. Provide a multi-agent epistemic axiomatisation  $T$  (translation) of  $O_1 \cup G_1$ ,  $O_2 \cup G_2$  and  $O_3 \cup G_3$  as knowledge and  $A'_{12}$  and  $A_{13}$  as beliefs.

| $T(G_2 \cup O_2)$                     | $T(G_1 \cup O_1)$                           | $T(G_3 \cup O_3)$                     |                                    |
|---------------------------------------|---|---------------------------------------|------------------------------------|
| $K_2o2:outdoor \sqsubseteq o2:sport$  | $K_1o1:dry \sqsubseteq o1:sport$            | $K_3o3:ground \sqsubseteq o3:sport$   |                                    |
| $K_2o2:indoor \sqsubseteq o2:sport$   | $K_1o1:wet \sqsubseteq o1:sport$            | $K_3o3:water \sqsubseteq o3:sport$    |                                    |
| $K_2o2:outdoor \perp o2:indoor$       | $K_1o1:dry \perp o1:wet$                    | $K_3o3:ground \perp o3:water$         |                                    |
| $K_2o2:outdry \sqsubseteq o2:outdoor$ | $K_1o1:dryball \sqsubseteq o1:dry$          | $K_3o3:grdcoll \sqsubseteq o3:ground$ |                                    |
| $K_2o2:outwet \sqsubseteq o2:outdoor$ | $K_1o1:drynob \sqsubseteq o1:dry$           | $K_3o3:grdind \sqsubseteq o3:ground$  |                                    |
| $K_2o2:outdry \perp o2:outwet$        | $K_1o1:dryball \perp o1:drynob$             | $K_3o3:grdcoll \perp o3:grdind$       |                                    |
| $K_2o2:indry \sqsubseteq o2:indoor$   | $K_1o1:wetball \sqsubseteq o1:wet$          | $K_3o3:wtrcoll \sqsubseteq o3:water$  |                                    |
| $K_2o2:inwet \sqsubseteq o2:indoor$   | $K_1o1:wetnob \sqsubseteq o1:wet$           | $K_3o3:wtrind \sqsubseteq o3:water$   |                                    |
| $K_2o2:indry \perp o2:inwet$          | $K_1o1:wetball \perp o1:wetnob$             | $K_3o3:wtrcoll \perp o3:wtrind$       |                                    |
| $K_2o2:outdry(d:football)$            | $K_1o1:influences(d:football, d:handball)$  | $K_3o3:grdcoll(d:football)$           |                                    |
| $K_2o2:indry(d:handball)$             | $K_1o1:influences(d:handball, d:waterpolo)$ | $K_3o3:wtrind(d:natation)$            |                                    |
| $K_2o2:inwet(d:natation)$             | $K_1o1:influences(d:natation, d:waterpolo)$ | $K_3o3:wtrcoll(d:waterpolo)$          |                                    |
| $T(A'_{12})$                          |   | $T(A_{13})$                           |                                    |
| $B_2o1:sport \sqsupseteq o2:sport$    | $B_1o1:sport \sqsupseteq o2:sport$          | $B_1o1:sport \sqsupseteq o3:sport$    | $B_3o1:sport \sqsupseteq o3:sport$ |
| $B_2o1:sport \sqsubseteq o2:sport$    | $B_1o1:sport \sqsubseteq o2:sport$          | $B_1o1:sport \leq o3:sport$           | $B_3o1:sport \sqsubseteq o3:sport$ |
| $B_2o1:dry \sqsupseteq o2:outdry$     | $B_1o1:dry \sqsupseteq o2:outdry$           | $B_1o1:dry \sqsupseteq o3:ground$     | $B_3o1:dry \sqsupseteq o3:ground$  |
| $B_2o1:wet \sqsupseteq o2:inwet$      | $B_1o1:wet \sqsupseteq o2:inwet$            | $B_1o1:wet \sqsupseteq o3:water$      | $B_3o1:wet \sqsupseteq o3:water$   |
| $B_2o1:drynob \sqsupseteq o2:outwet$  | $B_1o1:drynob \sqsupseteq o2:outwet$        |                                       |                                    |
| $B_2o1:dry \sqsupseteq o2:indry$      | $B_1o1:dry \sqsupseteq o2:indry$            |                                       |                                    |

22. Does  $T \models_{DEL} B_1o1:influences(d:handball, d:waterpolo) \wedge B_1o1:dry(d:handball) \wedge B_1o1:wet(d:waterpolo)$ ?

Yes. Because  $K_1o1:influences(d:handball, d:waterpolo)$  entails  $B_1o1:influences(d:handball, d:waterpolo)$  and  $K_2o2:indry(d:handball)$  and  $B_1o1:dry \sqsupseteq o2:indry$  entails  $B_1o1:dry(d:handball)$  and finally  $K_3o3:wtrcoll(d:waterpolo)$  and  $K_3o3:wtrcoll \sqsubseteq o3:water$  entails  $K_3o3:water(d:waterpolo)$  which with  $B_1o1:wet \sqsupseteq o3:water$  entails  $B_1o1:wet(d:waterpolo)$ .

These last inferences are not correct. In fact, it would be necessary that Source 1 endorses  $G_2$  and  $G_3$  for this to hold (e.g.  $\forall \phi \in G_i, B_1\phi$ ).