Agent Ontology Alignment Repair through Dynamic Epistemic Logic

Line van den Berg  
Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LIG, F-38000 Grenoble, France  
line.van-den-berg@inria.fr

Manuel Atencia  
Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LIG, F-38000 Grenoble, France  
manuel.atencia@inria.fr

Jérôme Euzenat  
Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LIG, F-38000 Grenoble, France  
jerome.euzenat@inria.fr

ABSTRACT

Ontology alignments enable agents to communicate while preserving heterogeneity in their information. Alignments may not be provided as input and should be able to evolve when communication fails or when new information contradicting the alignment is acquired. In the Alignment Repair Game (ARG) this evolution is achieved via adaptation operators. ARG was evaluated experimentally and the experiments showed that agents converge towards successful communication and improve their alignments. However, whether the adaptation operators are formally correct, complete or redundant is still an open question. In this paper, we introduce a formal framework based on Dynamic Epistemic Logic that allows us to answer this question. This framework allows us (1) to express the ontologies and alignments used, (2) to model the ARG adaptation operators through announcements and conservative upgrades and (3) to formally establish the correctness, partial redundancy and incompleteness of the adaptation operators in ARG.

KEYWORDS

Ontology alignment; alignment repair; agent communication; dynamic epistemic logic

Reference:

1 INTRODUCTION

Agents use ontologies to represent their knowledge of the world. Generally, these ontologies are not the same. This causes a problem when agents try to communicate: how do the agents understand whether they are correct, complete or redundant? This question is part of the more general problem of facilitating interoperability between agents while preserving heterogeneity in their information. Ontology matching algorithms have been developed to allow agents with different knowledge representations, structured in ontologies, to communicate [22]. These aim to find relationships holding across entities of two ontologies, the ontology alignment.

Alignments are typically computed and provided as input to the agents before any communication or joint task occurs.

Ontology matching algorithms may output only partially correct or incomplete alignments. This means that, even if alignments are available, communication failures can still occur due to mistakes in the alignment. There have been several attempts to repair ontology alignments [1, 17, 21] that have been integrated with multi-agent systems via specific protocols [1, 19]. In these approaches, alignment repair is performed statically, i.e. independently of agent interaction. However, in some multi-agent scenarios, it is not realistic nor desirable for agents to stop interacting until the repair is completed. This is why several approaches to ontology matching have been proposed that attempt to dynamically repair alignments [2, 9, 10]. The Alignment Repair Game (ARG) [11–13] that is inspired by ideas of cultural language evolution [23] is one of them. In ARG, the agents are adaptive: they communicate and, in parallel, evolve alignments through local corrective actions whenever communication fails. This is achieved via adaptation operators that specify precisely how agents adapt the failing correspondence of the alignment.

ARG was evaluated experimentally and the experiments showed that the adaptive agents converge towards successful communication and improve their alignment [11, 13]. However, experiments alone are not sufficient to logically assess properties of operators; whether they are correct, complete or redundant. In this paper, we introduce a formal framework based on Dynamic Epistemic Logic [25] to answer this question. This contributes to (1) providing a formal framework for knowledge and belief evolution for logical agents in ARG, (2) formally defining correctness, redundancy and completeness of the adaptation operators, (3) theoretically comparing adaptive agents and logical agents and (4) defining new adaptation operators.

Yet, the scope of this theoretical framework is not limited to ARG: it can be extended to establish formal properties of other games that are designed for agents to improve and repair alignments through interaction. This would allow for a theoretical comparison of different dynamic matching algorithms.

In the remainder, we discuss the related work (§2) and provide the preliminaries (§3). We introduce DEOL (§4) and translate states of ARG to DEOL models (§5). The formal properties of the adaptation operators are then proved (§6). We conclude by emphasizing the contribution to the broader dynamic ontology matching field (§7).

2 RELATED WORK

Different techniques have been proposed to evolve alignments: gossiping amongst agents to reach global agreement [1], logical repair to enforce consistency [16, 18, 21] and prevention of logical
violations to agents’ ontologies via conservativity principles [17]. These have been integrated with multi-agent systems via specific protocols [1,19]. However, they are performed independently of agent tasks.

To overcome this problem, interaction-situated semantic alignment was proposed [2]. This is an ontology matching algorithm as framed by the interaction protocols used by agents to communicate. Alignments are induced depending on repeated successful interactions and failing interactions lead to revision. This proposal was further advanced to repair alignments through their use and generalized to less constrained protocols [9,10].

The Alignment Repair Game (ARG) [11] is inspired by cultural language evolution [23] to repair alignments through local corrective actions. Alignments are induced depending on repeated successful interactions whenever a communication failure occurs through application of the adaptation operators. The idea is that ultimately, by repeatedly playing ARG, the alignments converge towards better alignments.

**Definition 3.1 (Adaptation Operator).** An adaptation operator provides a strategy for agents to revise the failing correspondence of the alignment. It specifies, given the failure of the correspondence \( \langle C_a, C_b, R \rangle \) with \( R \in \{\subseteq,\sqsubseteq,\equiv,\} \) and failing object \( o \), what the agents should do. In [11,13] the following adaptation operators are introduced:

- **delete**: delete the correspondence from \( A_{ab} \);
- **replace**: replace the correspondence by \( C_a \equiv C_b \);
- **add**: in addition to replace, add the correspondence \( C_a^{\text{sup}} \supseteq C_b \) between \( C_a \) and the immediate superclass \( C_a^{\text{sup}} \) of \( C_a \);
- **addjoin**: in addition to replace, add the correspondence \( C_a^{\text{supO}} \supseteq C_b \) between \( C_a \) and the lowest superclass \( C_a^{\text{supO}} \) of \( C_a \) that is compatible with the object \( o \) (i.e. \( C_a^{\text{supO}}(o) \));
- **refine**: in addition to replace, add the correspondences \( C_a \sqsupseteq C_b^{\text{sub}} \) between \( C_a \) and all the subclasses \( C_b^{\text{sub}} \) of \( C_b \) that are not compatible with the object \( o \) (i.e. \( \neg C_b^{\text{sub}}(o) \));
- **refadd**: addjoin and refine.

We write \( \alpha_{\langle C_a, C_b, R \rangle}(o) \) for the application of adaptation operator \( \alpha \) to correspondence \( \langle C_a, C_b, R \rangle \) with failing object \( o \).

From the definition, every operator entails delete, the operators add, addjoin, refine and refadd entail replace and refadd entails addjoin and refine. Furthermore, the order of the actions that are performed by the adaptation operators does not matter. Figure 1 illustrates the effect of the adaptation operators.

**Definition 3.2 (Alignment Repair Game).** The Alignment Repair Game (ARG) is played by a set of agents \( \mathcal{A} \) with a common set \( D \) of objects. Each agent \( a \in \mathcal{A} \) is associated with an ontology \( O_a \) and a set \( \{A_{ab}\} \) of non-empty alignments is given between any two ontologies \( O_a \) and \( O_b \) that at least includes \( \top_a \equiv \top_b \). We write \( O_1 \) for the most specific class (\( \subseteq \)-wise) of object \( o \in D \) available in \( O_1 \).

At each round of the game:

1. Two agents \( a, b \in \mathcal{A} \) and an object \( o \in D \) are picked at random.
Agent Ontology Alignment Repair through Dynamic Epistemic Logic

An ARG state is the state of the alignments reached after a, possibly empty, sequence of rounds where in each failing round an adaptation operator is applied.

Definition 3.4 (ARG State). For an ARG game with a set $\mathcal{A}$ of agents, a set $\{O_i\}_{i\in\mathcal{A}}$ of ontologies and a set $\{A_{ij}\}_{i,j\in\mathcal{A}}$ of alignments, an ARG state $\{A_{ij}'\}_{i,j\in\mathcal{A}}$ is the set of alignments $A_{ij}'$ reached from $\{A_{ij}\}_{i,j\in\mathcal{A}}$ after a, possibly empty, sequence of rounds.

For an ARG state $s$, we also write $\alpha(C_a, C_b, R_i(o))$ for the result of applying the adaptation operator $a$ to $s$ with failing correspondence $C_a R_i C_b$ and object $o$, or simply $\alpha(s)$ when the correspondence and object are clear from the context.

Example 3.5 (Running Example). In case of a failure of the correspondence $\text{Black}_a \equiv \text{Small}_b$, an adaptation operator is applied, adding the following new correspondences to the alignment and deleting the initial correspondence:
- delete: none
- replace: $\text{Black}_a \equiv \text{Small}_b$
- add: $\top_a \equiv \text{Small}_b$
- add join: $\bot_a \equiv \text{Small}_b$
- refine: $\text{Black}_a \equiv (\text{Square}_b \sqcap \text{Small}_b)$
- ref add: $\bot_a \equiv \text{Small}_b$ and $\text{Black}_a \equiv (\text{Square}_b \sqcap \text{Small}_b)$

By playing ARG with different operators, they can be compared. In Euzenat [11, 13], the operators are compared experimentally in terms of success rate (ratio of successes over rounds played), semantic precision and recall with respect to the known correct reference alignment (the degree of correctness and completeness of the resulting alignment) and convergence (the number of rounds needed to converge). It was found that all the operators have a relatively high success rate, yet do not reach 100% precision, and that recall and convergence both increase with operators that add new correspondences. The operator ref add, followed by add, shows the highest semantic recall and replace, again followed by add, the slowest convergence.

3.3 Dynamic Epistemic Logic

Dynamic Epistemic Logics (DEL) are a family of modal logics describing information flow in multi-agent systems. DEL has been widely used as a formal framework to model agent communication [7, 20, 25], belief revision [6] and agent interaction [24]. As such, it provides a solid basis to study knowledge and belief evolution of logical agents playing ARG. Here we consider the syntax and semantics introduced by Baltag, Moss and Solecki [5].

Definition 3.6 (Syntax of DEL). The syntax, $\mathcal{L}_{\text{DEL}}$, of (multi-agent) DEL is defined in the following way:

$$\phi ::= p \mid \phi \land \psi \mid \neg\phi \mid K_a\phi \mid B_a\phi \mid [\top\phi]\psi$$

where $p \in \mathcal{P}$ is a proposition, $K_a$ and $B_a$ are the knowledge and belief operators for each agent $a$ and $\top$ with $\top \in \{!, !\}$ the dynamic upgrades.

The connectives $\lor$ and $\rightarrow$, and the duals $\bar{K}_a, \bar{B}_a, [\top\phi]$ are defined in the usual way: $\phi \lor \psi \iff \neg
\neg\phi \land \neg\psi$, $\phi \rightarrow \psi \iff \neg\phi \lor \psi$, $\bar{K}_a\phi = \neg K_a\neg\phi$, $\bar{B}_a\phi = \neg B_a\neg\phi$, and $\langle\top\phi\rangle = \neg[\top\phi]\neg\psi$. DEL models are based on Kripke frames with plausibility relations where the logical dynamics act as model transformers.
Definition 3.7 (DEL Model). A model of (multi-agent) DEL is a quadruple $\mathfrak{M} = \langle W, (\succeq_a)_{a \in A}, w^\#, V \rangle$ where

- $W$ is a non-empty set of states, or worlds;
- $(\succeq_a)_{a \in A} \subseteq W \times W$ are the plausibility relations on $W$, one for each agent, that are converse well-founded, locally connected preorders;
- $w^\# \in W$ is the actual world;
- and $V$ is a propositional valuation mapping propositions to sets of worlds in which that proposition is true.

The plausibility relation $w \succeq_a v$ reads as “$w$ is at least as plausible as $v$ for agent $a$” and the epistemic and doxastic relations are defined on $W$ accordingly:

$$w \sim_a v \iff w (| w |_a \cup \succeq_a) v$$

(1)

$$w \rightarrow_a v \iff v \in \text{Max}_{\succeq_a} | w |_a$$

(2)

where $| w |_a$ is the information cell (or accessible cell) of agent $a$ at state $w$ and is defined by:

$$| w |_a = \{ v \in W \mid w \sim_a v \}$$

(3)

It follows from the properties of $\succeq_a$ and $\succeq_a$ that the relations $\sim_a$ are reflexive, transitive and symmetric, and the relations $\rightarrow_a$ are transitive, serial and Euclidean. Therefore they satisfy the usual properties of knowledge and belief, respectively [8, 25].

Definition 3.8 (Semantics of DEL). The semantics for DEL is defined in the following way:

$$M, w \models p \iff w \in V(p)$$

$$M, w \models \phi \land \psi \iff M, w \models \phi \text{ and } M, w \models \psi$$

$$M, w \models \neg \phi \iff M, w \not\models \phi$$

$$M, w \models K_a \phi \iff \forall v. s.t. w \rightarrow_a v : M, v \models \phi$$

$$M, w \models B_a \phi \iff \forall v. s.t. w \rightarrow_a v : M, v \models \phi$$

$$M, w \models [\phi]_1 \psi \iff M^\phi, w \models \psi$$

$$M, w \models [\psi]_1 \phi \iff M^\psi, w \models \phi$$

where $\psi$ and $\phi$ act as model transformers $\emptyset : M \rightarrow M^\psi$ and $\emptyset : M \rightarrow M^\phi$ in the following ways, with $|\phi|_M = \{ w \in W \mid M, w \models \phi \}$:

- **Announcement ($\emptyset$)** Delete all $\neg \psi$-worlds from the model.

  $W^\emptyset = \{ |\phi|_M, w \succeq_a^\emptyset v \mid w \succeq_a v \text{ and } w, v \in W^\emptyset \}$

  $V^\emptyset(p) = V(p) \cap |\phi|_M$ and $(w^\#)^\emptyset = w^\#$.

- **Conservative upgrade ($\psi$)** Change the plausibility orders so that the best $\psi$-worlds become better than all other worlds, while the old ordering on the rest of the worlds remains. I.e., $w^\psi = W, w \succeq_a^\psi v$ if either $w \in \text{Max}_{\succeq_a} (|\psi|_M \cap |\phi|_M)$ or $w \succeq_a v, V^\psi(p) = V(p)$ and $(w^\#)^\psi = w^\#$.

We also write $\emptyset, \phi, \emptyset \psi$ for the sequence of upgrades $\emptyset, \phi$ and then $\emptyset \psi$. The resulting model $M^{\emptyset, \phi, \emptyset \psi}$ is equal to $(M^{\emptyset, \psi})^{\emptyset \psi}$.

The intuition behind the different upgrades is that the trustworthiness of the information source may vary: it may be considered from an infallible source (announcements), or from a trusted, but not infallible source (conservative upgrades). For this reason, conservative upgrades only change the plausibility of worlds without deleting any alternatives.

Note that in all cases, $w^\#$ remains the actual world of the model. This also means that an announcement $\emptyset \phi$ can only be validly performed on a model $M$ if $\phi$ is true there.

4 DYNAMIC EPISTEMIC ONTOLOGY LOGIC

To compare adaptive agents with logical agents, we need a logical framework to model ARG. Here, we introduce Dynamic Epistemic Ontology Logic (DEOL) that is a variant of Dynamic Epistemic Logic where the propositions are object classifications ($\mathcal{C}(x)$) and class relations ($\mathcal{C} \equiv D, \mathcal{C} \subseteq D$ and $\mathcal{C} \uplus D$) of a Description Logic language. This logic enables us to later capture knowledge and belief evolution in alignment repair.

Definition 4.1 (Syntax of DEOL). The syntax, $\mathcal{L}_{DEOL}$, of (multi-agent) DEOL is defined in the following way:

$$\phi ::= \mathcal{C}(o) \mid \mathcal{CRD} \mid \phi \land \psi \mid \neg \phi \mid K_a \phi \mid B_a \phi \mid [\psi] \psi$$

$$R \in \{\subseteq, =, \emptyset\}, \uparrow \in \{!, \uparrow\}$$

where $C, D, \top \in \mathcal{C}, o \in D, K_a$ and $B_a$ are the knowledge and belief operators for agent $a$ and $\hat{\uparrow}$ with $\hat{\uparrow} \in \{!, \uparrow\}$ are the dynamic upgrades.

The connectives $\rightarrow$ and $\land$ and the duals $K_a, B_a, [\hat{\uparrow}] \phi$ are defined as in the case of DEL.

DEOL models are plausibility models. The difference with DEL models is that instead of a valuation of propositions, we consider a model of interpretation $\Lambda$ representing the objects and an interpretation function $I$ assigning to each world a function interpreting each class as a set of objects of the domain.

Definition 4.2 (DEOL Model). A model of (multi-agent) DEOL is a quintuple $\mathfrak{M} = \langle W, (\succeq_a)_{a \in A}, w^\#, \Lambda, I \rangle$ where

- $W$ is the set of states, or worlds;
- $(\succeq_a)_{a \in A} \subseteq W \times W$ are the plausibility relations on $W$, one for each agent, that are converse well-founded, locally connected preorders;
- $w^\# \in W$ is the actual world;
- $\Lambda$ is the domain of interpretation (a set of objects);
- and $I$ is an interpretation function $s.t. I(w) = I_w : C \rightarrow \mathcal{P}(\Lambda)$, where it holds that $\top^{I_w} = \Lambda$, and for any two classes $C, D \in \mathcal{C}$ we have that $(C \cap D)^{I_w} = C^{I_w} \cap D^{I_w}$ and $(\neg C)^{I_w} = \Lambda \setminus C^{I_w}$ for each $w \in W$.

We also write $C \uplus D$ for the class defined by $\neg (\neg (C \cap D))$, and $\uplus \{C_1\}$ and $\cap \{C_1\}$ for the classes defined by $C_1 \cap C_2 \cap \ldots$ and $C_1 \cup C_2 \cup \ldots$, respectively. Their interpretations at world $w$ are given by $C^{I_w} \uplus D^{I_w}, \cap C^{I_w}$ and $\cup C^{I_w}$, respectively. In each DEOL model $\bot = \neg \top$ is the empty class.

The semantics of DEOL is equivalent to that of DEL except that we now have instance classifications $\mathcal{C}(o)$ and class relations $C \subseteq D$, $C \equiv D$ and $C \uplus D$. 
Agent Ontology Alignment Repair through Dynamic Epistemic Logic

Definition 4.3 (Semantics of DEOL). The semantics for DEOL extends that of DEL (Definition 3.8) by:

\[ M, w \models_c (\phi) \iff \phi^M \in C^M \]
\[ M, w \models_c C \iff C^M \subseteq D^M \]
\[ M, w \models_c C \equiv D \iff C^M = D^M \]
\[ M, w \models_c C \otimes D \iff C^M \cap D^M = \emptyset \]

The additional capacities of logical agents compared to the original game are that logical agents can now use the relations between concepts to reason about instance classification. For instance, the following axiom schemata are valid:

\[ K_a(C(x)) \land K_a(C \subseteq D) \models K_a(D(x)) \quad (4) \]
\[ K_a(C(x)) \land K_a(C \oplus D) \models K_a(\neg D(x)) \quad (5) \]

In addition, agents can combine their knowledge and beliefs to obtain new beliefs. For instance, \( K_a(C_a(o)) \land B_a(C_a(o)) \) entails \( B_a(C_a(o)) \). In other words, agent \( a \) can transfer some of her knowledge about \( C_a \) to beliefs about \( C_b \).

This increased reasoning capacity of logical agents compared to adaptive agents is crucial in the results later about the correctness, partial redundancy and incompleteness of the adaptation operators.

5 TRANSLATION

In the previous section, we have provided a formal framework for knowledge and belief evolution in alignment repair. Now we use this framework to capture the Alignment Repair Game. More precisely, we define a translation from ARG states to DEOL axioms that are interpreted as sets of DEOL models (§5.1) and from adaptation operators for ARG to dynamic upgrades on DEOL (§5.2), see Figure 3. These translations are labeled by \( z \) and \( \delta \), respectively, and the interpretation on DEOL models by \( I \). This enables us to define and prove correctness, redundancy and completeness of the adaptation operators in the remainder of this paper.

\[
\begin{array}{c}
\text{ARG state (s)} \xrightarrow{z} \text{DEOL axioms} \xrightarrow{I} \text{DEOL models (M)} \\
\text{ARG state (s) } \xrightarrow{\delta} \text{DEOL axioms} \xrightarrow{I} \text{DEOL models (M\Phi(\delta))}
\end{array}
\]

Figure 3: Diagram of translations from ARG states to DEOL axioms (\( z \)) that are interpreted by sets of DEOL models (\( I \)), and from adaptation operators to dynamic upgrades (\( \delta \)).

5.1 Semantics of ARG states

Let agents \( a \) and \( b \) play ARG with ontologies \( O_a \) and \( O_b \), respectively, and alignment \( A_{ab} \). Given the nature of ontologies and alignments, we impose the following three conditions on the DEOL axioms describing the epistemic-doxastic states of agents \( a \) and \( b \):

Ontology Knowledge (OK) \( O_a \) (\( O_b \)) is known to agent \( a \) (\( b \));
Alignment Belief (AB) \( A_{ab} \) is believed by agents \( a \) and \( b \);
Public Signature Awareness (PSA) The signatures of all ontologies are known to all agents.

In the interpretation on DEOL models, this means that the sentences that describe \( O_a \) are true in any world in \( |w|_a \), and the sentences that describe \( A_{ab} \) are true in all most plausible worlds in both \( |w|_a \) and \( |w|_b \).

Example 5.1 (Running Example). In the running example, this means that the sentences \( \langle \text{Square}_a \equiv \text{Small}_a \rangle \subseteq \text{Small}_a \), \( \langle \text{Small}_a \equiv \text{T}_{\text{r}} \rangle \), \langle T_{\text{r}} \equiv \text{T}_{\text{b}} \rangle \) are true in every accessible world for agent \( a \) and that the sentences \( \text{Small}_a \equiv \text{Black}_a \) and \( \text{T}_{\text{r}} \equiv \text{T}_{\text{b}} \) are true in the most plausible worlds for both agents.

Public signature awareness ensures that agents are allowed to update their information when we consider the dynamics of the adaptation operators. It requires that, for each agent \( a \), each object \( o \in D \) and for each two classes \( C, D \in O_b \) with \( b \neq a \) and not appearing in the alignment, i.e. \( C, D \notin \{ C_b \in O_b \mid \langle C_a, C_b \rangle \in A_{ab}, C_a \in O_a \} \), agent \( a \) considers all combinations of the following alternatives equally plausible:

- \( C(o) \) and \( \neg C(o) \)
- \( D(o) \) and \( \neg D(o) \)
- \( C \equiv D \) and \( C \not\equiv D \)

Formally, this is achieved on the interpretation on DEOL models by ultimately making as many copies of the worlds describing the agent’s knowledge and belief as there are combinations of the alternatives above, ranking them all equally plausible while respecting the order imposed by the alignments.

Because models rapidly explode, we will only draw the information given by the ontologies and alignments.

Example 5.2 (Running Example). Figure 4 depicts the epistemic-doxastic state of agent \( a \) at the start of the game. Note that the alignment \( A_{ab} \) consisted of \( \langle \text{T}_{\text{r}} \equiv \text{T}_{\text{b}} \rangle \) and \( \langle \text{Black}_a \equiv \text{Small}_b \rangle \).

Note that the interpretation of the DEOL translation of ARG with \( O_a, O_b \) and \( A_{ab} \) satisfying OK, AB and PSA is not unique. Indeed, there are many variations of models that qualify, and, in particular

\[ \text{minimal DEOL model } M^\text{min}_{O_a, O_b, A_{ab}} \] (or \( M_{\text{min}} \) in short when it is clear from the context) in which agents have no other knowledge or beliefs than given by the closure of the three conditions.

Proposition 5.3. Any DEOL model \( M \) describing ARG with \( O_a, O_b \) and \( A_{ab} \) that satisfies the three conditions is an extension of the minimal DEOL model \( M^\text{min}_{O_a, O_b, A_{ab}} \).
5.2 Dynamics

During the gameplay of ARG, new information is learned. There are two dynamic acts involved in the learning: the communication of $C_b(o)$ in step 2 of ARG and the adaptation operator applied in step 5 (see Definition 3.2). How do these acts change the knowledge and beliefs of the agents? And are the adaptation operators as defined by Euzenat [11, 13] sufficient to account for these changes?

In order to answer these questions, we translate the communication taking place in ARG to dynamic upgrades on DEOL.

**Definition 5.4 (ARG Dynamics in DEOL).** We model each round of ARG as defined in Definition 3.2 by

$$!C_b(o); \text{ if } -C_a(o) \text{ then } \delta(a(C_a, C_b, \triangleright)(o))$$

where $\delta(a(C_a, C_b, \triangleright)(o))$ denotes the translation of adaptation operator $a$ applied to the correspondence $C_a \sqsubseteq C_b$ with failing object $o$.

Given that $C_b(o)$ is knowledge to agent $b$, the communication of this information in step 3 of ARG translates to an announcement on DEOL. For the adaptation operators, announcements are not the correct tool: adaptation operators tell the agents how to revise the alignment, their beliefs, upon a communication failure. Therefore adaptation operators translate to conservative upgrades.

**Definition 5.5 (Adaptation Operators as Dynamic Upgrades).** Let $\langle C_a, C_b, \sqsupseteq \rangle \in A_{ab}$ be the failing correspondence with object $o$, the adaptation operators $\langle a(C_a, C_b, \sqsupseteq)(o) \rangle$ are translated to the following dynamic upgrades on DEOL (where the subscript $\langle C_a, C_b, \triangleright\rangle(o)$ is left out for readability):

$$\delta(\text{delete}) = \uparrow(C_a \sqsubseteq C_b)$$

$$\delta(\text{replace}) = \uparrow(C_a \sqsupset C_b)$$

$$\delta(\text{add}) = \uparrow(C_a \sqsubseteq C_b \land C_a^{\text{supO}} \sqsubseteq C_b)$$

$$\delta(\text{addjoin}) = \uparrow(C_a \sqsubseteq C_b \land C_a^{\text{supO}} \sqsubseteq C_b)$$

$$\delta(\text{refine}) = \uparrow(C_a \sqsubseteq C_b \land \bigcup\{C_a \sqsubseteq C_b^{\text{supO}}\})$$

$$\delta(\text{refadd}) = \uparrow(C_a \sqsubseteq C_b \land C_a^{\text{supO}} \sqsubseteq C_b \land \bigcup\{C_a \sqsubseteq C_b^{\text{supO}}\})$$

where $C_a^{\text{sup}} = \text{Min}_{\sqsupseteq}\{C \in O_a \mid C_a \sqsubseteq C\}$, $C_a^{\text{supO}} = \text{Min}_{\sqsupseteq}\{C \in O_a \mid C_a \sqsubseteq C \land C(o)\}$ (by construction of the ontologies, $C_a^{\text{sup}}$ and $C_a^{\text{supO}}$ are unique) and $\{C_a \sqsubseteq C_b^{\text{supO}}\} = \{C_a \sqsubseteq C \mid C \in O_b \land C \sqsubseteq C_b \land C(o)\}$. If the initial correspondence of the alignment is an equivalence-relation, i.e. if $\langle C_a, C_b, \equiv \rangle \in A_{ab}$, then the corresponding dynamic upgrade for delete is $\uparrow(C_a \sqsubseteq C_b \land C_a \sqsubseteq C_b)$. The upgrades for the other adaptation operators remain the same.

Again, as was the case for the adaptation operators on ARG states, $\delta(\text{add}), \delta(\text{addjoin}), \delta(\text{refine})$ and $\delta(\text{refadd})$ entail $\delta(\text{replace})$, and $\delta(\text{refadd})$ entails $\delta(\text{addjoin})$ and $\delta(\text{refine})$.

**Example 5.6 (Running Example - Success).** When ARG is played with $\blacktriangle$, agent $b$ announces $\text{!Small}_{b}(\blacktriangle)$ and the correspondence used is $\langle \text{Black}_{a}, \text{Small}_{b}, \equiv \rangle \in A_{ab}$. This information is compatible with the information of agent $a$: $\text{Black}_{a}$ is compatible with $\text{Small}_{b} \cap \text{Black}_{a}$, i.e. the most specific class of $\blacktriangle$.

Compared to ARG where the round is now finished, there are additional epistemic-doxastic changes on the corresponding DEOL model. The announcement carries more information than just indicating that the round of ARG was a success, it transforms some beliefs of agent $a$ into knowledge: $B_{a}(\text{Small}_{b}(\blacktriangle))$ becomes $K_{a}(\text{Small}_{b}(\blacktriangle))$. In other words, agent $a$ is now given concrete evidence that $\blacktriangle$ is a member of $\text{Small}_{b}$ whereas she only believed this. Figure 4 can be compared to and the upper schema of Figure 5 for an overview of the changes to the epistemic-doxastic state of agent $a$.

![Figure 5: The knowledge (solid black) and belief (dashed red) of agent $a$ of Example 3.3 after the announcement $\text{!Small}_{b}(\blacktriangle)$ (above) and after the announcement $\text{!Small}_{b}(\blacktriangle)$ (below).](image-url)

**Example 5.7 (Running Example - Failure).** If instead ARG is played with $\blacktriangledown$, the round is a failure. Agent $b$ announces $\text{!Small}_{b}(\blacktriangledown)$ using the same correspondence $\langle \text{Black}_{a}, \text{Small}_{b}, \equiv \rangle \in A_{ab}$. However, this information contradicts the knowledge of agent $a$ and, as a result, the correspondence (belief) of the alignment will be dropped.

However, this is not the only revised belief. The contradicted initial beliefs turn into knowledge of their negation. For example, $B_{a}(\neg \text{Small}_{b}(\blacktriangledown))$ becomes $K_{a}(\text{Small}_{b}(\blacktriangledown))$ after the announcement. Compare also Figure 4 and the lower schema of Figure 5 for an overview of the changes to the epistemic-doxastic state of agent $a$.

According to ARG, an adaptation operator is applied, which results in an updated alignment as explained in Example 3.5. These correspondences should be amongst the beliefs of the agents at the end of the round of ARG. However, for some operators, the correspondences are already believed by agent $a$ before the adaptation operator is applied. We will see why in the next section.

The translation provided in this section is faithful because the semantics of ontologies and alignments we use is the same as in Description Logic and the only dynamic epistemic component arises...
from modeling the agents’ knowledge and beliefs, see also [4]. A formal proof is out of the scope of this paper.

6 FORMAL PROPERTIES OF THE ADAPTATION OPERATORS

With the formal representation of ARG in DEOL we can explore the correctness, redundancy and completeness of the operators. For this, we consider the diagram as pictured in Figure 3.

6.1 Correctness

To show that the adaptation operators are correct, we need to show that the diagram of Figure 3 commutes.

Definition 6.1 (Correctness). Adaptation operator \( \alpha \) is correct if and only if \( \forall s: (z(s))^{\delta(\alpha)} \models z(\alpha(s)) \).

Proposition 6.2. The adaptation operators delete, replace, addjoin, refine and readd are correct.

Proof. We do the proof for agent \( a \) and adaptation operator addjoin. The proof for replace now follows because it is entailed by addjoin, and the proof for refine is symmetric.

Because addjoin only adds beliefs, it suffices to show that these are entailed: \( (z(s))^C_{K_a(\alpha)}(\delta(\alpha))/C_{K_a(\alpha)}(\delta(\alpha)) \models B_I(C_a \equiv C_b) \land B_i(C_a \equiv C_b) \). This holds because initially the correspondence is believed, i.e. \( z(s) \models B_I(C_a \equiv C_b) \), and the upgrade \( U_k(\alpha) \); \( (C_a \equiv C_b \land C_{sup} \equiv C_b) \) deletes all the worlds from \( z(s) \) in which \( C_o(\alpha) \) is false and then rearranges the remaining worlds such that the \( C_a \equiv C_b \land C_{sup} \equiv C_b \)'-worlds become more plausible than the \( \neg(C_o(\alpha) \land C_{sup} \equiv C_b) \)'-worlds. Because there remain \( C_a \equiv C_b \land C_{sup} \equiv C_b \)'-worlds accessible for both agents, the belief is enforced. For agent \( b \), this is true because the announcement \( U_k(\alpha) \) does not alter her epistemic-doxastic state (she already knew that \( C_o(\alpha) \) as it is in her ontology), and for agent \( a \), because the announcement \( U_k(\alpha) \) deletes the worlds in which \( C_o(\alpha) \) holds because the correspondence and announcement caused a failure) or \( C_o(\alpha) \) but not those in which \( C_a \equiv C_b \land C_{sup} \equiv C_b \). Therefore the beliefs \( B_i(C_a \equiv C_b) \) and \( B_i(C_{sup} \equiv C_b) \) are enforced for agents \( i \in \{a, b\} \). Hence addjoin is correct. \( \square \)

Yet, the adaptation operator add is not correct because it does not take into account whether the immediate superclass of \( C_a \) is consistent with the object \( o \). And if it is consistent, add is equivalent to addjoin.

Proposition 6.3. The adaptation operator \( \alpha = add \) is incorrect, i.e. \( \exists s: (z(s))^{\delta(\alpha)} \not\models z(\delta(\alpha)) \), and \( \forall s.t. (z(s))^{\delta(\alpha)} \models z(\delta(s)): add(s) = addjoin(s) \).

Proof. We need to prove the existence of an ARG state \( s \) where \( (z(s))^{\delta(\alpha)} \not\models z(\delta(\alpha)) \) with upgrade \( \delta(\alpha) = U_k(\alpha) \); \( (C_a \equiv C_b \land C_{sup} \equiv C_b) \), object \( o \); \( O_a \models C_o(\alpha) \) and \( C_a \equiv C_b, \equiv \in A_{ab} \) the failing correspondence. Pick \( s \) to be any such ARG state where the immediate superclass \( C_{sup} \) of \( C_a \) is incompatible with \( o \), i.e. \( O_a \not\models C_{sup} \) \( (o) \). Then \( z(s) \models K_a(-C_{sup}(o)) \) and \( (z(s))^{\delta(\alpha)} \models K_a(C_o(\alpha)) \land K_a(-C_{sup} \equiv C_b) \). This is because \( \delta(\alpha) \) deletes all \( -C_b(o) \)'-worlds from \( z(s) \) and therefore all the worlds accessible by agent \( a \) for \( C \equiv C_b \) for \( C \) such that \( z(s) \models K_a(C(o)) \). In particular, this holds for \( C_{sup} \). But, after applying the adaptation operator add, \( C_{sup} \equiv C_b \) becomes part of the alignment, so that \( z(\delta(\alpha)) \models B_a(C_{sup} \equiv C_b) \). Hence \((z(s))^{\delta(\alpha)} \not\models z(\delta(s)) \).

Moreover, whenever \( (z(s))^{\delta(\alpha)} \models z(\delta(\alpha)) \) it must be that \( O_a \models C_{sup} \equiv C_b \) so that, per definition, \( C_{sup} = C_{sup} \), i.e. add is equivalent to addjoin. \( \square \)

Proposition 6.3 is in line with initial predictions and experimental results by Euzenat [13, 14]: addjoin shows faster convergence than add. This is because add can force false correspondences to be added to the alignment that can later cause a failure. From these results, it is clear that for a logical agent, add should be abandoned.

6.2 Redundancy

The redundancy of some operators in the running example is not a coincidence. For logical agents, i.e. DEOL agents, some adaptation operators are redundant for every ARG state: delete, replace and addjoin are redundant with respect to agent \( a \) and refine is redundant with respect to agent \( b \). Before we define this redundancy with respect to one agent (partial redundancy), let us first consider what it means for an operator to be redundant (with respect to both agents). An adaptation operator \( \alpha \) is redundant if and only if solely applying \( U_k(\alpha) \) on the DEOL translation of \( s \) is already sufficient to obtain an interpretation of the DEOL translation of \( \alpha(s) \).

Definition 6.4 (Redundancy). Adaptation operator \( \alpha \) is redundant if and only if \( \forall s: (z(s))^{\delta(\alpha)} \models z(\alpha(s)) \).

\[
\begin{array}{c}
\text{ARG state (s)} \\
\delta \quad \text{DEOL axioms} \\
\alpha \quad \text{DEOL models (M)} \\
\end{array}
\]

\[
\begin{array}{c}
\text{ARG state (s)} \\
\alpha \quad \text{operator} \\
\text{DEOL models (M^φ(a))} \\
\end{array}
\]

Figure 6: Diagram of translations between ARG states DEOL axioms that are interpreted on DEOL models, adaptation operators and dynamic upgrades as in Figure 3 where the operators are redundant.

The adaptation operators discussed here are not redundant, but partially redundant. This means that they are redundant with respect to one agent. To prove redundancy, we show that the knowledge and belief of this agent is invariant to the application of the adaptation operator. In fact, because adaptation operators only alter the beliefs of agents, it suffices to show partial redundancy by showing that the beliefs of that agent remain unchanged.

Definition 6.5 (Partial Redundancy). An adaptation operator \( \alpha \) is partially redundant for agent \( a \) if and only if \( (z(s))^{\delta(\alpha)} \models B_a φ \) implies \( z(\alpha(s)) \models B_a φ \) for each ARG state \( s \) and each \( φ \) in \( L_{DEOL} \).

Proposition 6.6. The adaptation operators delete, replace and addjoin are partially redundant with respect to agent \( a \), and refine is partially redundant with respect to agent \( b \).
PROP. We do the proof for agent \( a \) and the adaptation operator addjoin. The proof for replace now follows because it is entailed by addjoin, and the proof for refine is symmetric.

Thus we need to show that \( (z(s))^C_b(o) \models B_a \phi \) implies that \( z(\text{addjoin}_{C_a, C_b}(s))(s) \models B_a \phi \). So consider a sentence \( \phi \) that is not believed by agent \( a \) in \( z(\text{addjoin}_{C_a, C_b}(s))(s) \), but is in \( z(s) \). By construction of the dynamics of the operator addjoin, this can only be (1) a belief that is inconsistent with \( C_b(o) \) (because the announcement \( \Diamond C_b(o) \) deletes these worlds), or (2) \( C_a \not\subseteq C_b \) (because it is enforced by the conservative upgrade part of the dynamics). But these are also not believed by agent \( a \) in \( z(s) \), (1) because \( C_b(o) \) has deleted all these beliefs, and (2) because \( z(s) \models K_a(\neg(C_b(o))) \) and this knowledge is invariant under the announcement \( \Diamond C_b(o) \), causing the belief in \( C_a \not\subseteq C_b \) to be dropped.

Hence, by contraposition, addjoin is partially redundant with respect to agent \( a \). In Figure 7 the knowledge and belief of agent \( a \) is illustrated before and after the announcement \( \Diamond C_b(o) \) for an intuition.

The incompleteness proof of the adaptation operators relies on the agent not memorizing the failure of the correspondence with the drawn object. Yet, from Figure 7 it is clear that there is more knowledge gained by the agents from the announcement \( \Diamond C_b(o) \). This occurs because we measure completeness, and correctness, with respect to the full knowledge and belief of the agent. When concentrating on the alignment only, as expressed by adaptive agents, the operators may be complete.

6.3 Incompleteness

Finally, we consider completeness of the adaptation operators: do the operators capture all the information that can be learned? Intuitively, this is proven by comparing what is learned by the agents in ARG scenarios from application of the adaptation operators with what is learned by logical agents in DEOL from the dynamic upgrades. If the former implies the later, the operator is (epistemically) complete.

Definition 6.7 (Completeness). Adaptation operator \( a \) is complete if and only if \( \forall s: z(\alpha(s)) \models (z(s))^\delta(\alpha) \).

Proposition 6.8. All adaptation operators (delete, replace, add, addjoin, refine, refadd) are incomplete.

Proof. Again, consider the knowledge and belief of agent \( a \) before and after the announcement \( \Diamond C_b(o) \), see also Figure 7. After the announcement \( \Diamond C_b(o) \), agent \( a \) receives concrete information that object \( o \) belongs to the class \( C_b \), i.e. she comes to know this information: \( (z(s))^C_b(o) \models K_a(C_b(o)) \). And, by definition, this knowledge remains after application of any conservative upgrade, i.e. \( (z(s))^\delta(\alpha) \models K_a(C_b(o)) \). Yet, this knowledge is never acquired through application of the adaptation operators because they only concern the alignment, i.e. beliefs of class relations, and not knowledge of instance classification. Hence \( z(\alpha(s)) \not\models K_a(C_b(o)) \) and \( z(\alpha(s)) \not\models (z(s))^\delta \).

7 DISCUSSION AND CONCLUSION

We developed a theoretical framework for knowledge and belief evolution in ARG and formally defined correctness, completeness and redundancy of adaptation operators to compare adaptive agents and logical agents. We complement the current experimental approach by proving that, in this framework, all but the add operator are correct, delete, replace, addjoin and refine are redundant for one agent and that all operators are incomplete. This contributes to theoretically comparing the different operators and could inspire new adaptation operators for ARG. However, this does not mean that the adaptive agents are meaningless. In fact, the adaptive agents in [11, 13] implement deliberately a ‘sublogical’ behavior and the experiments show that, in some cases, they can perform well. For instance, agents do not need to be fully complete, and not even fully correct, to reach 100% success in ARG. The purpose of our work is to examine them under a logical light. We compare adaptive agents to logical agents, and we prove the logical limitations of the adaptation operators.

Yet, the scope of this paper lays beyond ARG. We have provided a theoretical framework that can be extended to establish formal properties of other games that are designed for agents to improve and repair alignments through interaction. This allows for a theoretical comparison of different dynamic matching algorithms.

In this paper, public signature awareness was a prerequisite in the translation from ARG states to DEOL to capture the dynamics of the game by announcements and conservative upgrades. In the future, we want to drop this prerequisite. We suspect that this might provide the means to capture the ability to generate new (random) correspondences [13].

ACKNOWLEDGMENTS

The authors thank the anonymous reviewers for their valuable comments and helpful suggestions. This work has been partially supported by MIAI @ Grenoble Alpes (ANR-19-P3IA-0003).
REFERENCES


