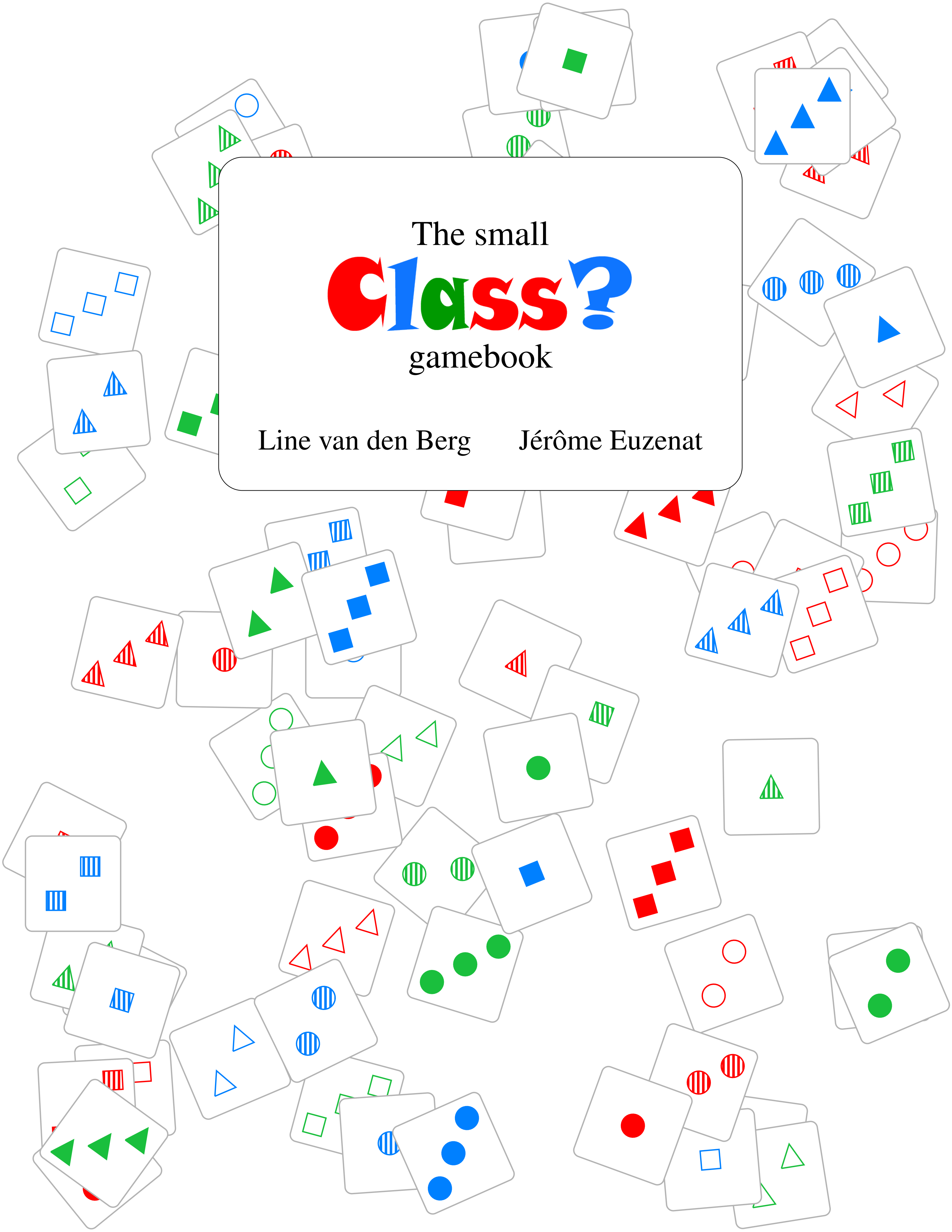

The small Class? gamebook

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Abstract

Class? is an enjoyable card game aiming at grouping colourful cards into meaningful classes. It illustrates facets of reasoning with classifications. In order to introduce *Class?* progressively, this *small gamebook* provides a sequence of games before getting to the *Class?* game itself and beyond. The games are presented in increasing order of difficulty so that a game will benefit from mastering of previous ones.

https://moex.inria.fr/mediation/class/

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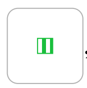
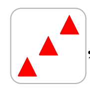
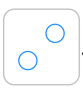
Introduction

Class? is a card game that we introduced in order to illustrate our work in knowledge representation and evolution. It is based on a set of 81 cards, all different, representing a pattern. This pattern is determined by four characteristics that each takes exactly one value among three. Those are:

– the number: 1, 2 or 3 elements,
– the color: red, green or blue,
– the shape: square, triangle or round,
– the filling: empty, striped or filled.

For example, one striped green square , three filled red triangles , or two blue empty circles .

Although *Class?* is itself fun to play it requires to master some simple and useful notions with which not everybody is familiar. These are the notion of classes of cards defined by shared characteristics, order relations between such classes, etc. Such notions are worth introduced independently from the *Class?* game and this may be done using the *Class?* material. This can be done in an entertaining and intriguing way.

Moreover, we noted that the material available to us to teach important ideas of computing were very often based on set theory and arithmetics, but were not very well suited to draw the interest on partial orders, yet a very useful tool.

Hence we designed this *small Class? gamebook* as a collection of games which use the *Class?* material and introduce progressively one of these concepts at a time. Some of these games are real games that may provide a lot of fun and others are mere exercises illustrating important techniques. They also can be used as a demonstration to open a discussion.

This small book is addressed at those who would like to initiate others to these simple games and concepts. It tries to provide information useful for doing so. All games are played with the *Class?* deck of cards and sometimes with classification boards. They may require some pens, papers and the template presented in Appendix A.

The games are (totally) ordered so that each one relies on mastering of the previous one.

Each game is systematically described along the following outline:

Audience the targeted audience;
Duration the durations of the exercise;
Resources the material support needed;
Learning outcomes the goal of the current exercise;
Prerequisites the skills that are required to do the exercise;
Skills which skills are learned through this exercise;
Game play the rules of a game;
Example an example of how the game is played;

Variations some different ways to play the game;

Science how the game is used to teach the learning outcomes;

Questions the questions that may be asked to the players after playing (questions are rated by *, **, *** from the easy to the more difficult ones);

Thoughts to be shared what this exercise is supposed to teach/illustrate (things to wrap up after playing the game);

Knowing more/doing more references to other resources.

Audience age are indicative. Duration must be understood as the time taken for someone who has mastered the previous games to start mastering the current one.

Appendix B provides short answers to the questions.

Do not hold your breath too long, there is no plan for a big book. You'd better try this one now.

Acknowledgements. The development of *Class?* has been supported by INRIA.

The *Computer science unplugged* project[1] (or tangible coding in another style) obviously inspired this game collection. We also took inspiration from the page of Marie Duflot-Kremer[2] for the description of exercises.

[1] https://www.csunplugged.org/.

[2] https://members.loria.fr/MDuflot/files/med/index.html

Game 1

What is a class?

1.1 Audience

Ages 8 and up.

1.2 Duration

10 minutes.

1.3 Resources

- Cards.

1.4 Learning outcomes

- Learn what a class is and how to describe it;
- Learn that cards (objects) may be grouped together into classes, defined by a membership relation;
- Learn that there are different classes to which the same card may belong, there is no unique class.

1.5 Prerequisites

- Know how to describe the characteristics of the cards: shape (circle, square and triangle), number (1, 2 and 3), color (green, blue and red) and texture (filled, striped, empty).

1.6 Skills

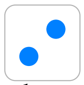
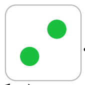
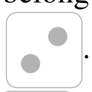
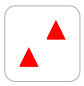
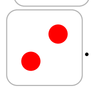
- Analyzing differences and similarities.

1.7 Game play

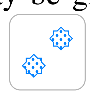
1. Pick two *Class?* cards that are different on up to three characteristics (color, shape, texture or number).
2. Ask what the cards have in common and what is different between them.

3. Describe the smallest class to which they belong: it is defined by what they have in common.
4. Pick two other cards, one which shares the common characteristics and one which does not, and ask whether they belong to it or not.
5. For the card that does not belong: can we describe the class to which it belongs, together with the other cards? How?
6. Repeat the operation.

1.8 Example

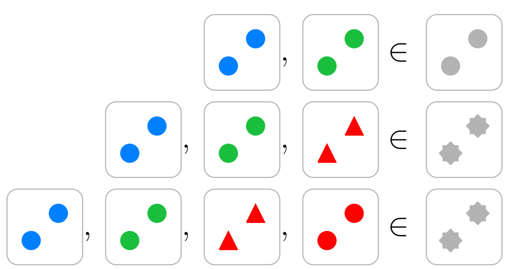
1. Pick two cards  and .
2. They have their shape (circle), number (2) and texture (filled) in common, but their color is different (blue and green).
3. The smallest class to which these belong is the class described by all the cards with two filled circles. This can be indicated as follows: .
4. Pick two other cards  and . The second card belongs to that class but the first card does not.
5. The class to which all the cards belong together is the class described by cards with two filled figures. This can be indicated by: .
6. Repeat.

1.9 Science

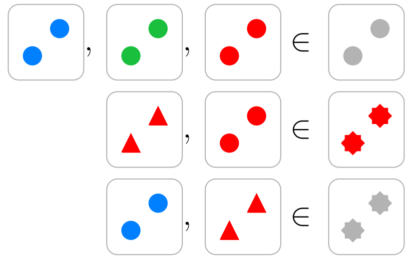
Cards may be grouped in classes, representing a set of cards with specific characteristic values. For example,  represents the class of all cards having two blue elements. To denote that the shape and the filling are not specified for this class neutral shapes and patterns, that cannot be found on any of the cards (here star and dotted), are used. These can be replaced by any value (square, triangle or round; empty, striped or filled).

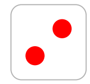
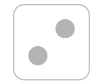
For any set of cards, there is a class to which they belong together. Appendix A provides another way to conveniently represent classes.

The example teaches the following membership equations (noting by $\in$ that cards belong to a class):

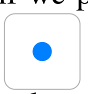
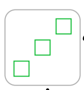
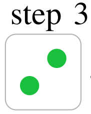
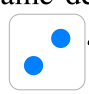


but also (with the same cards):



We learn that there are different classes to which a card belongs. For example,  belongs to  but also to  and .

1.10 Questions

1. * Does each belong to a unique class?
2. ** Does each belong to a unique smallest class?
3. ** How many different classes are there?
4. ** What if we pick cards that do not share any characteristic (color, shape, texture or number), for instance  and ? Is there a class to which they both belong?
5. *** Is the class chosen in step 3 of the game dependent on the order of picking cards? Is there a difference if we first pick  and then ?

1.11 Thoughts to be shared

- Cards may be grouped in different ways to form classes, there is no unique way.
- But the smallest class is unique, it is the class described by the characteristics that the cards have in common.
- Even cards with no similar characteristic belong to a common class: the class of all cards.
- The order is irrelevant to determine the class of the cards.

1.12 Knowing more/doing more

- Try Card splitting classes.

Game 2

Card splitting classes

2.1 Audience

Ages 8 and up.

2.2 Duration

15 minutes.

2.3 Resources

– Cards.

2.4 Learning outcomes

– Learn how to create a discriminating class.

2.5 Prerequisites

– Know what a class is (What is a class?).

2.6 Skills

– Analyzing differences and similarities.

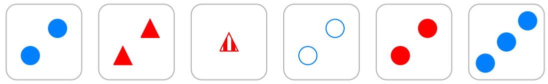
2.7 Game play

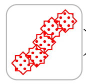
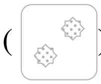
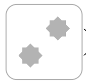
1. 6 cards are laid out in front of the players.
2. They each choose a class such that three of their cards belong to it and the three others do not belong to it. If it happens that no such class can be found, two further cards from the deck.
3. The first player to provide such a class wins.

   Players can play in turn.

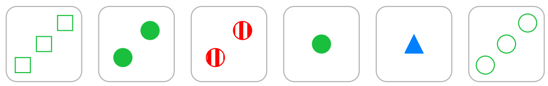
2.8 Example

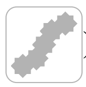
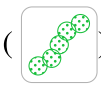
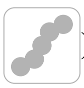
A first set of cards is laid out on the table as:



Player 1 proposes red cards (), Player 2 proposes cards with two elements (), Player 3 proposes cards with two filled elements (). Players 1 and 3 are correct, but not Player 2 since there are 4 cards with two elements.

A second set of cards is drawn as:



Player 1 proposes filled cards (), Player 2 proposes green circles (), Player 3 proposes filled circles (). Players 1 and 2 are correct, but not Player 3 since there are only two filled circles.

2.9 Science

With respect to the previous game, 'Card splitting classes' introduces the capability to exclude cards from a class.

So, instead of defining a class through the common characteristics, it is also possible to define them through the non common ones.

Trying to find a class discriminating between two sets of cards requires to isolate those characteristic value combinations which applies to only three of the cards. The task is difficult because there are already a large combination of sets of three cards.

2.10 Questions

1. * How many combinations of sets of three cards are there?
2. ** Is it always possible to find such a class?
3. ** If it is possible, is the class necessary unique?

2.11 Thoughts to be shared

– A class determines the set of cards belonging to it as well as the set of those non belonging to it (the others).

2.12 Knowing more/doing more

– Try Guessing classes.
– You could have a look at the SET! game[1], in which the goal is to find a class of three cards out of 9.

[1] https://setgame.com

Game 3

Guessing classes

3.1 Audience

Ages 12 and up.

3.2 Duration

15 minutes.

3.3 Resources

- Cards;
- Support for noting classes (see Appendix A).

3.4 Learning outcomes

- Learn how to guess a class from a set of examples and counter-examples (examples to show the opposite).

3.5 Prerequisites

- Knowing how to create a splitting class.

3.6 Skills

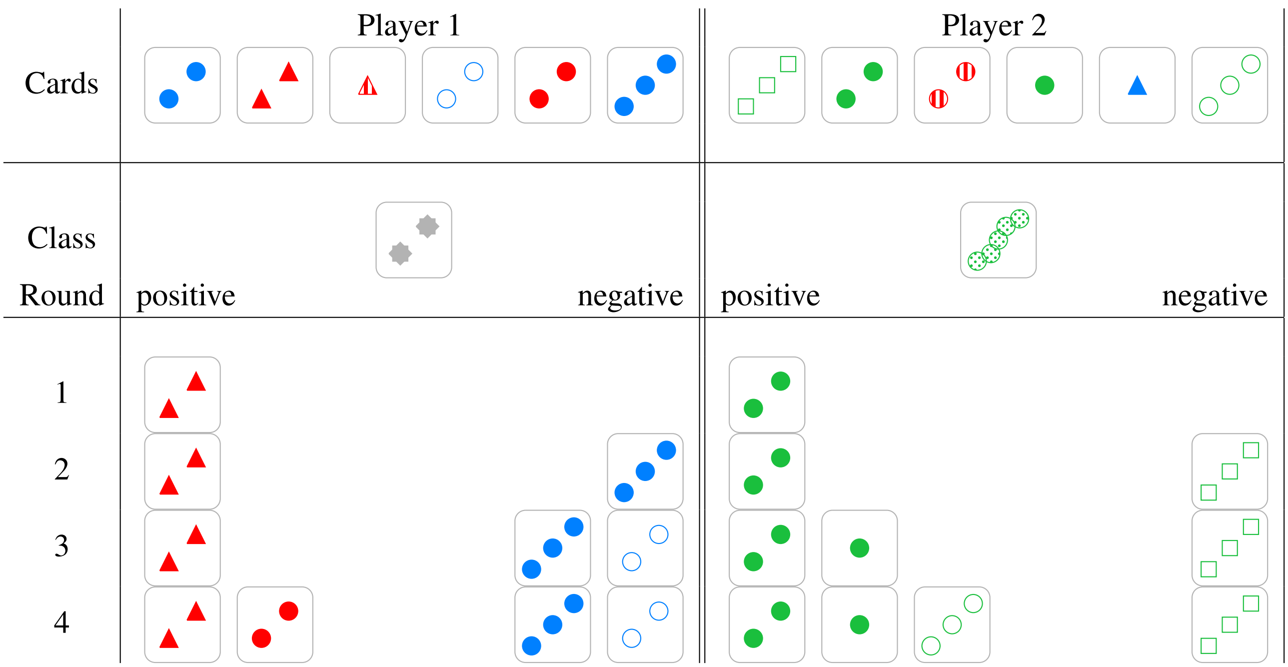
- Analyzing differences and similarities.

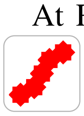
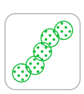
3.7 Game play

1. Each player receives 6 cards.

2. They each choose a class such that three of their cards belong to it and the three other ones do not belong to it (splitting class). They keep their class secret (see Appendix A). If it happens that no such class can be found, they can draw two further cards from the deck.
3. One by one, the players ask the other player for a positive (belonging to the class) or negative (not belonging to the class) card for the chosen class of the other player.
4. If the player has no card for the asked category (positive or negative), the player takes cards from the deck and assigns them to the piles until the player finds a card that is asked for (positive or negative).
5. The first player to guess the other's class wins.

3.8 Example



At Round 4, Player 2 suggests that Player 1's class is the class of cards with red filled elements (), which is wrong. At Round 4, Player 1 finds that Player 2's is the class of cards with green circles (), which is correct.

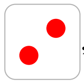
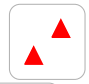
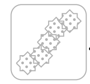
3.9 Variations

– It is possible to play with the cards visible and the player asks for each card if they are positive or negative.
– The card to be disclosed may also be taken randomly, i.e. not asking for positive or negative.

These variations should allow for less strategy. On the contrary, if the players can choose any card to give as example, this can be used strategically (and may last longer).

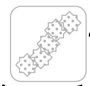
3.10 Science

With respect to the previous game, 'Guessing classes' requires to keep track of the examples in order to establish the possible classes covering the positive examples and excluding the negative ones.

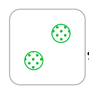
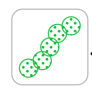
For instance, , belongs to many different classes. Adding  (as another positive example) would determine several common classes such as , , , and .

Said differently, each example separates the set of all possible classes into two sets: those classes to which it belongs and those to which it does not belong. The former is selected if the example is positive; the latter if it is negative. Hence, a set of positive and negative examples selects the set of classes to which all positive example belong and none of the negative example belong.

3.11 Questions

1. ** How many cards are necessary to determine the class of all cards, ?
2. ** Do the examples and counter-examples always correspond to a unique class?
3. *** How many cards do we need to play to be sure to have determined the class?

3.12 Thoughts to be shared

- Classes can be guessed from examples and counter-examples.
- A set of examples and counter-examples may correspond to several classes: the cards of Player 2 in the example that belong to the class may describe the classes ,  or .

3.13 Knowing more/doing more

- This can be understood as a variation of the Mastermind game (in which this is not the player that proposes the result). Can you provide a variation closer to Mastermind?
- Try Subclasses.

Game 4

Subclasses

4.1 Audience

Ages 8 and up.

4.2 Duration

20 minutes.

4.3 Resources

- Cards;
- (Possibly) support for noting classes (see Appendix A).

4.4 Learning outcomes

- Learn how classes relate to each other and may be minimally extended.

4.5 Prerequisites

- Know what a class is (What is a class?).

4.6 Skills

- Relations between classes.
- Transitivity of the relations.
- Minimality.

4.7 Game play

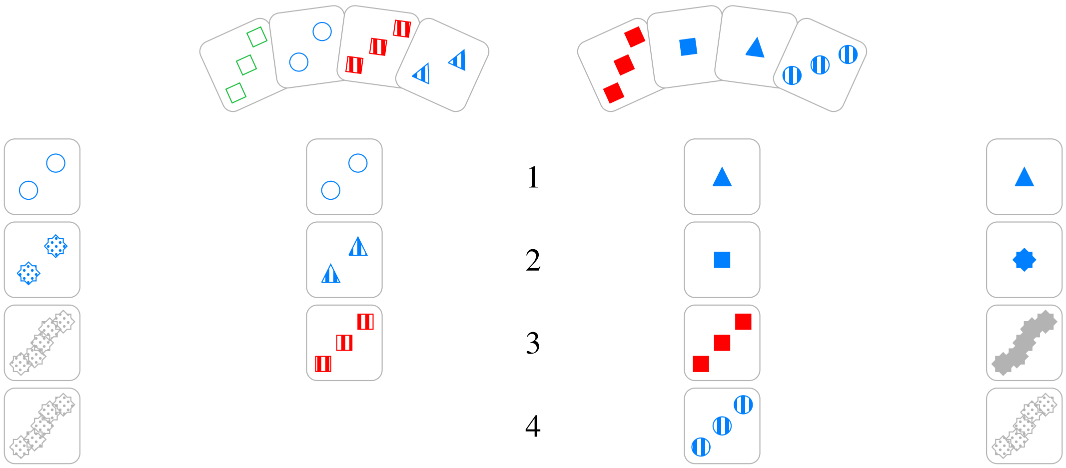
1. Each player is given five cards which are not shown to the others;

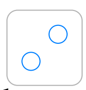
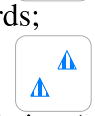
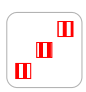
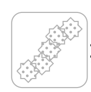
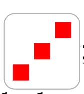
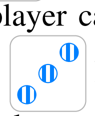
2. Each player in turns puts one card in front of her so that the class corresponding to all the cards on the table is larger than what it was before;
3. The game stops when no player can put a card on the table;
4. The player with less cards in her hands win.

During this game it is worthwhile that players tell the class they define aloud.

4.8 Example

The picture displays the four cards that each player has and, progressively, how they put them on the table. On the exterior, is displayed the corresponding class.



1. The left-hand player puts down her card  and the right-hand player ; The corresponding class is the one containing only these cards;
2. The left-hand player puts down her card  and the right-hand player ; in the former case, this extends the class by two characteristics (shape and filling); in the latter, only shape has been extended;
3. The left-hand player put down her card  leading to the class of all cards: ; the right-hand player plays ; this extends the class by two characteristics (color and number);
4. The left-hand player cannot further extend its class which already covers all cards; the right-hand player can play  leading to the class of all characteristics;
5. The right-hand player wins: she has no card anymore.

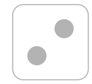
The subtlety of the game consists of extending classes in a minimal way so that more cards can be played. The cards must be played in a relevant order.

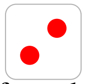
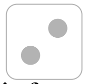
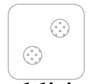
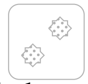
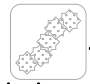
4.9 Variations

– It is possible to start with more cards than 5. This will introduce more strategy.

4.10 Science

This game introduces an important relation between classes: subclass.

We say that  is a subclass of (or a more specific class than)  and note it as follows:  $\sqsubseteq$ .

This is very general in fact,  $\sqsubseteq$  $\sqsubseteq$  $\sqsubseteq$  $\sqsubseteq$ .

When a class is a subclass of another, it freezes additional characteristic values over the other, and its elements (cards) are elements of the other.

4.11 Questions

1. * Are each pair of classes in subclass relations?
2. * Is there a class that is in subclass relation with all other classes?
3. ** What other relations between classes can you think of? Give examples.

4.12 Thoughts to be shared

– Classes may be related by a subclass relation, where the superclass is the generalization of the subclass, and the subclass a specialization of the superclass.
– These relations reflect both the definition of classes (they have increasingly selective) or the inclusion of the set of cards they describe.

4.13 Knowing more/doing more

– Consider Disjoint classes.

Game 5

Disjointness

5.1 Audience

Ages 12 and up. This game is more challenging to understand than the previous ones and even the two next ones.

5.2 Duration

30 minutes.

5.3 Resources

- Cards;
- (Possibly) support for noting classes (see Appendix A).

5.4 Learning outcomes

- Learn how classes may be specified positively and negatively.

5.5 Prerequisites

- Know what a subclass is (Subclasses).

5.6 Skills

- Relations between classes.
- Computing intersection.

5.7 Game play

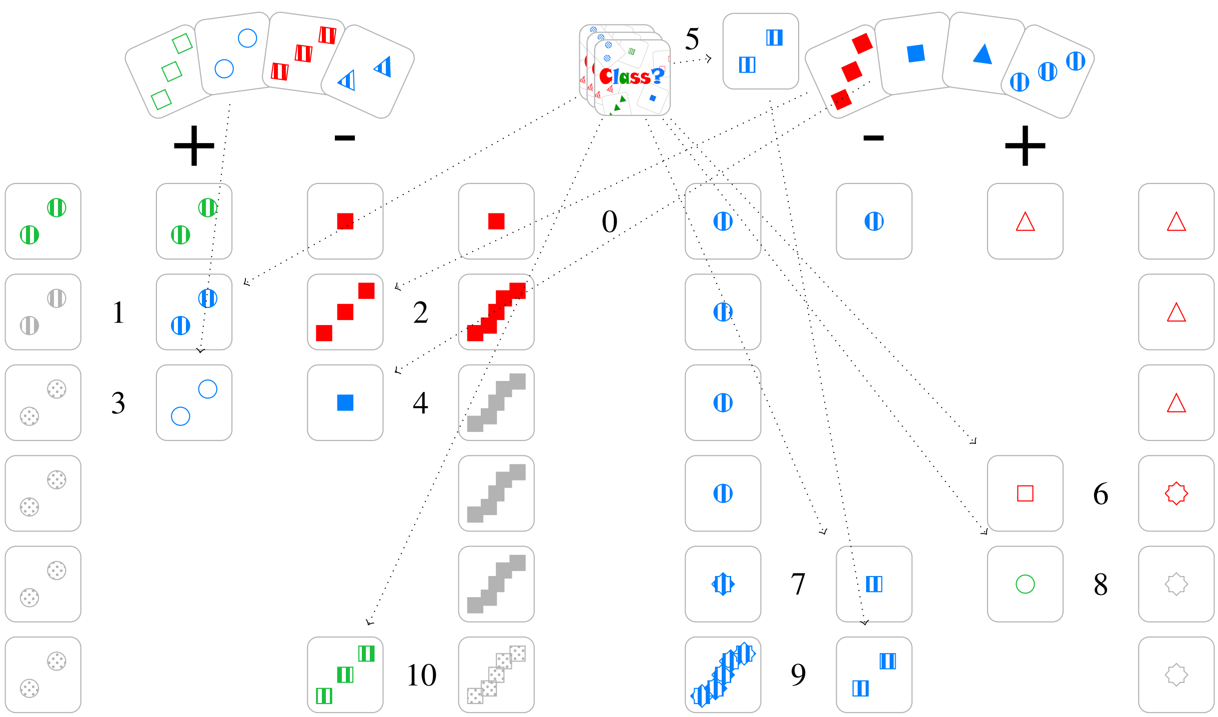
1. Each player is given four cards which are not shown to the others;

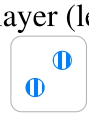
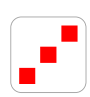
2. Each player has two cards on the table: one corresponds to the positive (+) class; the other to the negative (-) class;
3. Each player, taking turns:
   – either, adds one card to his or her positive class;
   – or, adds one card to another player's negative class;
   – or, draws one card from the deck and either plays one of the above actions, or adds it to his or her hand.

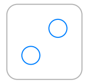
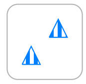
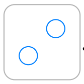
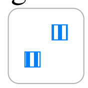
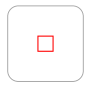
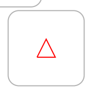
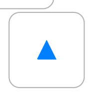
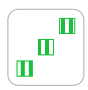
   A card is added to a class only if (*a*) it belongs to the class, or (*b*) it extends the class by one single characteristic and there is no card belonging to both the negative and the positive class.
4. The game ends at the end of the first round in which one of the players cannot expand his or her class or in which the deck is empty. This happens when one class has three indifferent characteristics, sometimes before. It is not possible to have four because this would necessarily cover cards in the negative class.
5. The player with the larger class, in terms of cards belonging to the class wins. In case of ex-aequo, the one with the least cards in her hand wins.

5.8 Example

The picture displays the initial situation (0) and for each turn the added card and the class corresponding to each sets of cards.



1. The first player (left) cannot add a card to either her positive class or the other player's negative class. She draws  from the deck and puts it in her positive class.
2. The second player (right) cannot add to his positive class, but can add  to the negative class of the other player, which he does.

3. The first player now has the choice to add  or  to his positive class. He decides to add .
4. The second player (right) still cannot add to his positive class, but can add  to the negative class of the other player, which she does.
5. The first player cannot anymore add  to his positive class. Indeed, the resulting class  would cover the two filled blue squares now belonging to the negative class. He cannot either add anything to the other player negative class. He draws the card or  from the deck that he cannot use. Hence, he keeps it in his hand.
6. The second player cannot play anything, he draws  from the deck which she may either add to his positive class or the other's negative class, he chooses the former.
7. The first player draws  from the deck that he adds to the second player's negative class.
8. The second player draws  from the deck that he can add to his positive class, but this does not change the class.
9. Now the first player can play  in the negative class of the second player.
10. The second player cannot play  to his positive class because the resulting class  would cover  which belongs to the class! She thus draws  from the deck, that he can play to the negative class of the first player.

Now it is clear that the first player's negative class cannot be extended anymore: it covers all square cards (one third of the cards). Its positive class is the class of all cards containing two circles. It still can be extended, to all circles (but not to all cards with two elements which would cover cards of the negative class).

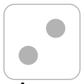
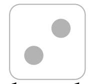
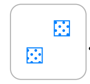
On the second player's side the positive class contains all cards with one single elements and empty filling and the negative all blue stripped cards. They both can be extended (for the positive by having arbitrary numbers and for the negative side by having arbitrary colors).

5.9 Variations

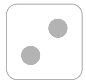
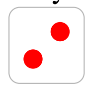
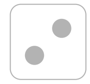
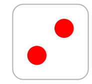
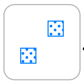
– It is possible to start with empty positive and negative classes: that would probably be more fun and more strategic.
– It may be possible to play several cards at once at each run, still with the same constraints.
– The winner may also be the first to finish.

5.10 Science

This game brings together disjointness and subclass.

We say that  is disjoint from  and note it as follows:  $\perp$ .

When in disjointness relation, no element of one class belongs to the other.

There are connections between disjointness and subclass: For any classes C, C' and C'', if $C \sqsubseteq C'$ and $C' \perp C''$ then $C \perp C''$. For instance, from  $\perp$  and  $\sqsubseteq$  one can deduce  $\perp$ .

5.11 Questions

1. * Are classes containing one single card necessarily disjoint (have no card in common)?
2. ** Could both players have the same positive class in the end?

5.12 Thoughts to be shared

– Not all classes are related via subclasses, there are pairs of classes that are different but have overlap or that have no card in common (this is disjunction).
– It is possible to expand two classes under the constraint that they remain disjoint.

5.13 Knowing more/doing more

– Try Identifying with classifications.

Game 6

Identifying with classifications

6.1 Audience

Ages 8 and up.

6.2 Duration

15 minutes.

6.3 Resources

- Cards;
- Classification boards (either printed or generated on computer).

6.4 Learning outcomes

- Learn how to find the class of a card in a classification correctly, and if possible efficiently.

6.5 Prerequisites

- Know what a class is (What is a class?).
- Know what a classification is (Subclasses).

6.6 Skills

- Applying a procedure.
- Matching an instance to a class.

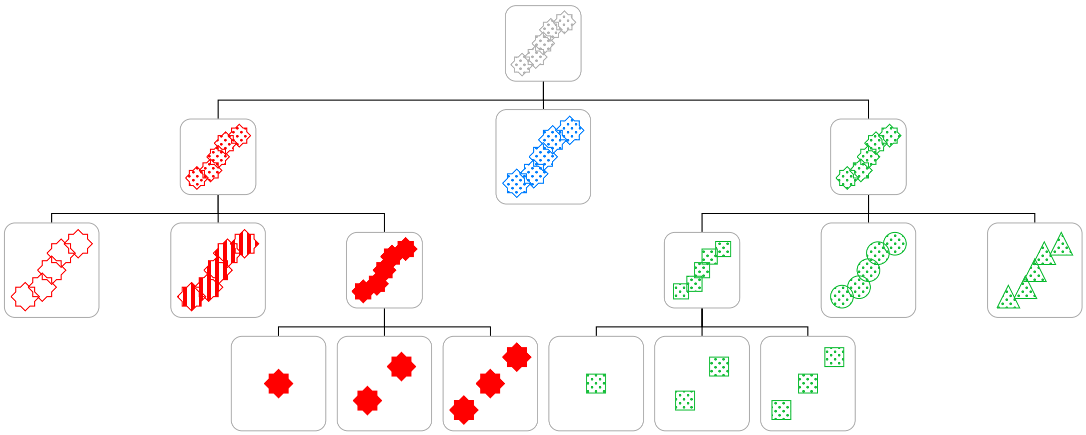
6.7 Game play

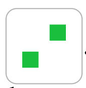
1. One classification is shown openly;
2. One player draws a card from the deck;

3. The other player asks, in turn, questions among ‘which colour is the card’, ‘what shape is the card?’, ‘which filling is the card?’, ‘how many elements are there in the card?’
4. The first player able to tell to which leave of the classification the card belongs wins.

6.8 Example

Here is the classification to be used:



A player draws the card .

1. One asks ‘what colour is the card?’, the player answers ‘green’.
2. Another one asks ‘what shape is the card?’, the player answers ‘square’.
3. Yet another one asks ‘how many elements are there on this card?’, he answers ‘two’. Then this player says: it belongs to class .

6.9 Variations

– It is possible to change the game so that the questions become ‘is the card red?’, ‘does the card has two elements?’, ‘are the elements of the card filled?’, etc. This variant makes the game less predictable, but it does not really teaches the identification procedure.
– Each player may have a hidden classification. These players will ask questions in turns. The first one to have reached a leaf class in her classification wins. Of course, they can take advantage of the answers to the questions of the others.

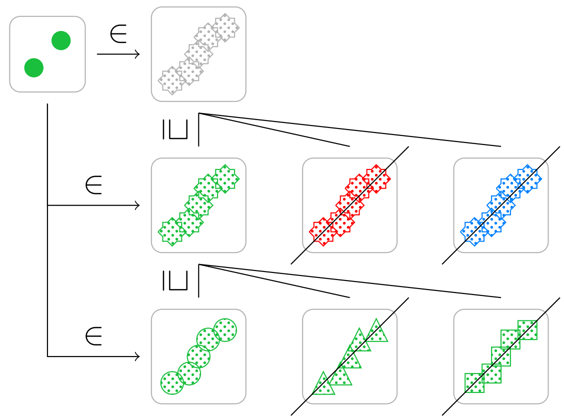
6.10 Science

Classes may be organized into a classification: a tree of classes (growing from top to bottom) like the one below, in which $\sqsubseteq$ is expressed by having the subclass under its superclass.

Each fork divides all the cards belonging to the class into three subclasses according to a single characteristic: the number, the color, the shape or the filling. The top class contains all the cards. And each class contains fewer cards than the class that precedes it and all the class cards that follow it (from top to bottom). On the same level of the tree, the criteria may be different.

In such a classification, each card belongs to one and only one leaf class (class without subclass) also called its most specific class.

The game is teaching how to proceed step by step to find the most specific class of a card in a given classification, using the subclass relation. In the example above that is:



The number of decisions to take (here 2) is far smaller than the number of classes in the classification.

6.11 Questions

1. * What is the maximum number of steps to make to reach a leave?
2. * Can there be two cards classified in a leaf class?
3. ** Does the card always belong to the same class in all classifications?

6.12 Thoughts to be shared

- Classes can be organized into a classifications via subset relations.
- In different classification, cards can belong to different classes. But in one classification, there is a unique class to which the card belongs.
- There is a procedure for finding the most specific class of a card (start from the top and choose the correct branch each time to find the most specific class).
- It is very similar to games that one would play on a holiday trip (Is it red? Is it an animal? Does Lucy have one?).

6.13 Knowing more/doing more

- To test an interesting application of this, compare with the '20 questions' game (http://www.20q.net/ or in French: https://fr.akinator.com).
- You are now ready to try the *Class?* game.
- However, if you want to know how it is possible to create classifications, you may try Building classifications.

Game 7

Building classifications

7.1 Audience

Ages 10 and up.

7.2 Duration

20 minutes.

7.3 Resources

- Cards;
- Support for drawing classifications (paper and pen). This game requires the ability to unambiguously draw a classification.

7.4 Learning outcomes

- Learn how relations between classes can be used to form full classifications.
- Learn that simple procedures may be used for developing full classifications from data.

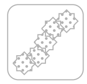
7.5 Prerequisites

- Know how classes may be related (Subclasses).
- Identify the class in a classification to which a card belongs (Identifying with classifications).

7.6 Skills

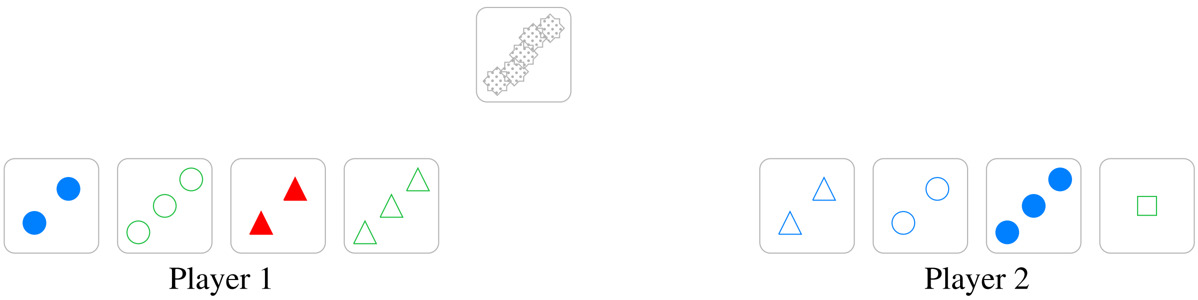
- Applying a procedure.
- Relations between classes.
- Transitivity of relations.

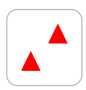
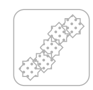
7.7 Game play

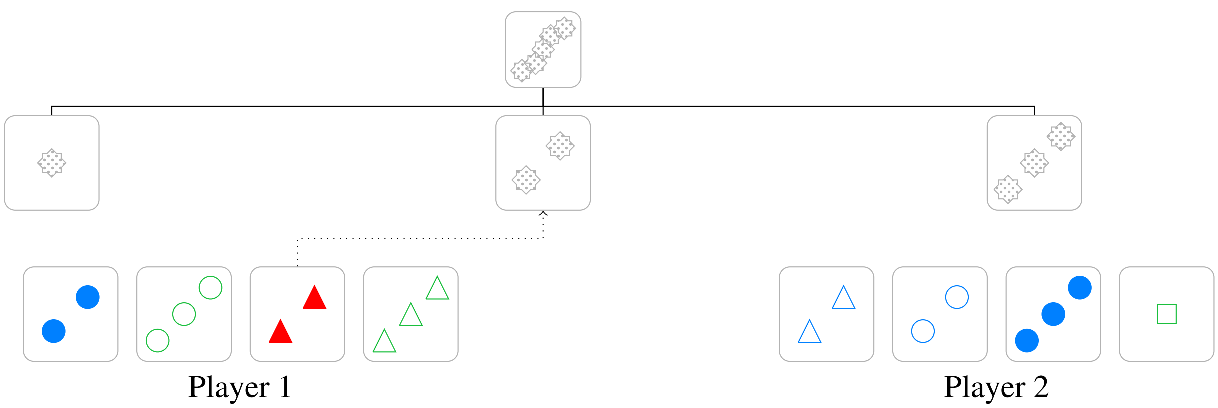
1. Each player has 4 cards, visible to all players.
2. Initially, the classification is reduced to its top: 
3. Each player taking turns:
   - can assign a card to a leaf class in the classification, only if no card is assigned to it, or
   - can expand one leaf class of the classification to which a card is assigned, by deciding the characteristic along which this class is further split. The assigned card is reassigned to the new leaf class it belongs to.
4. The first to assign all his or her cards wins.

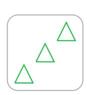
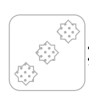
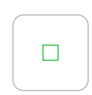
7.8 Example

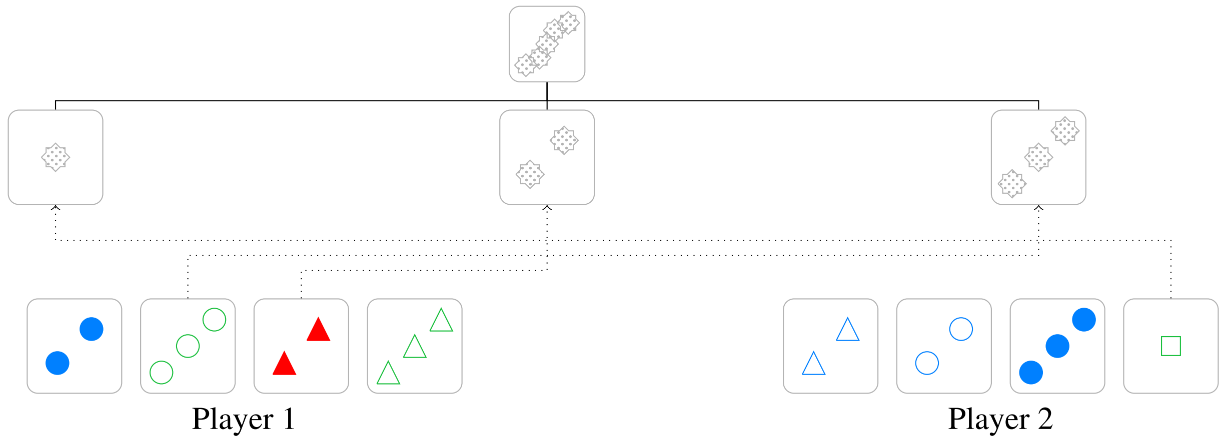
The initial situation: each player has four cards; the classification only has its top level.

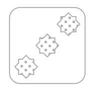


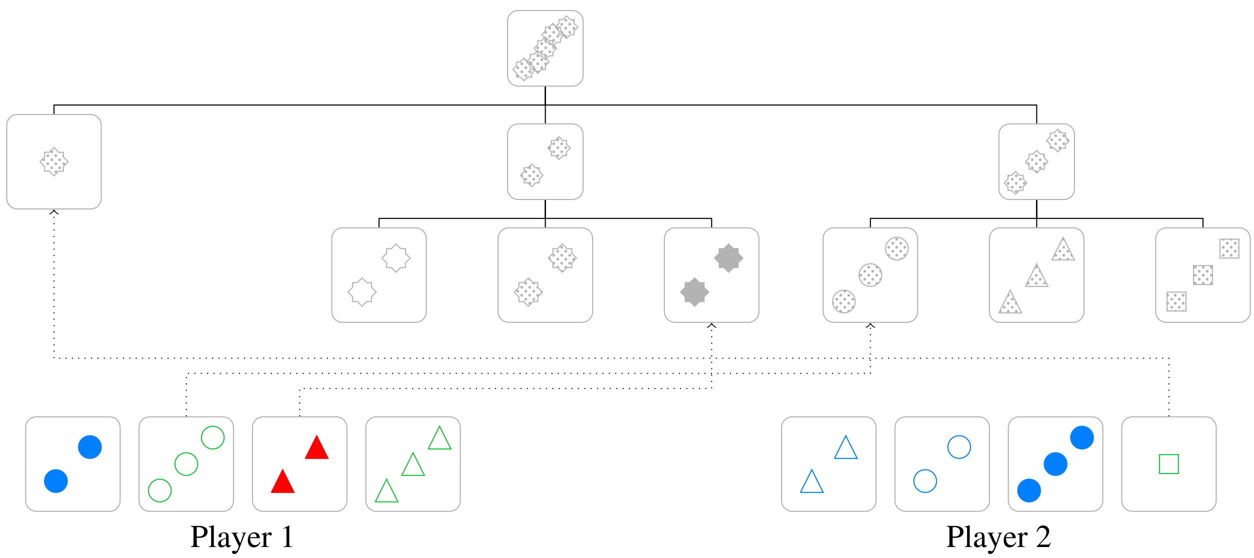
First round: Player 1 assigns  to the top class. Player 2 decides to discriminate cards in  by number. This gives the result:

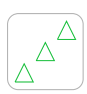
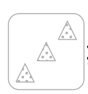
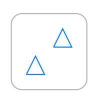
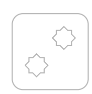


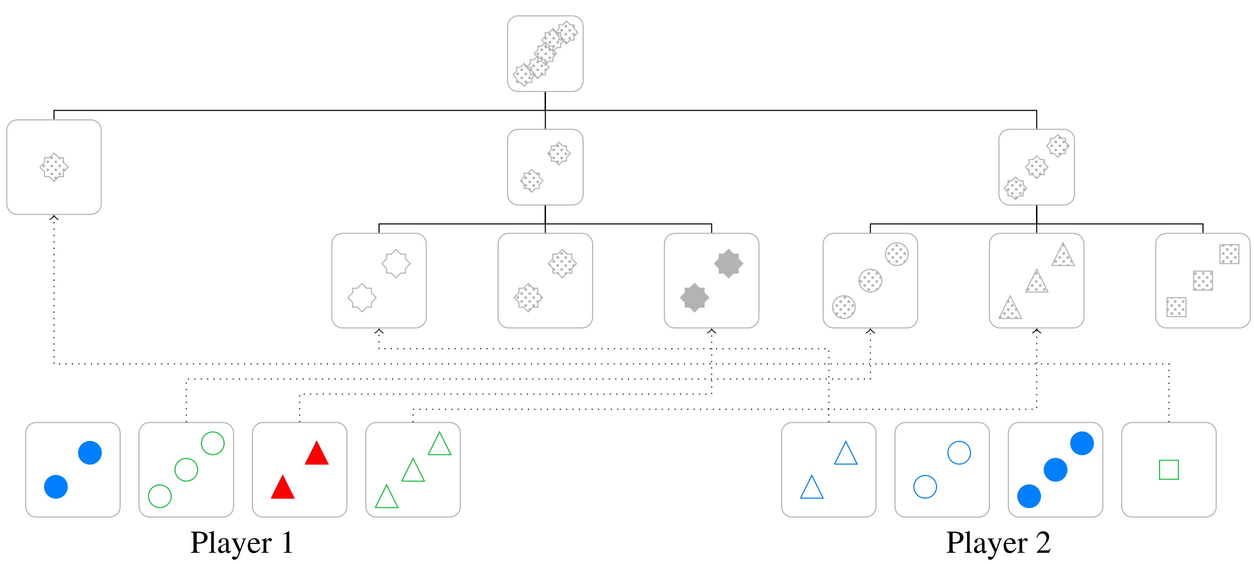
Second round: Player 1 attaches card  to class ; Player 2 attaches card  to class . This leads to:



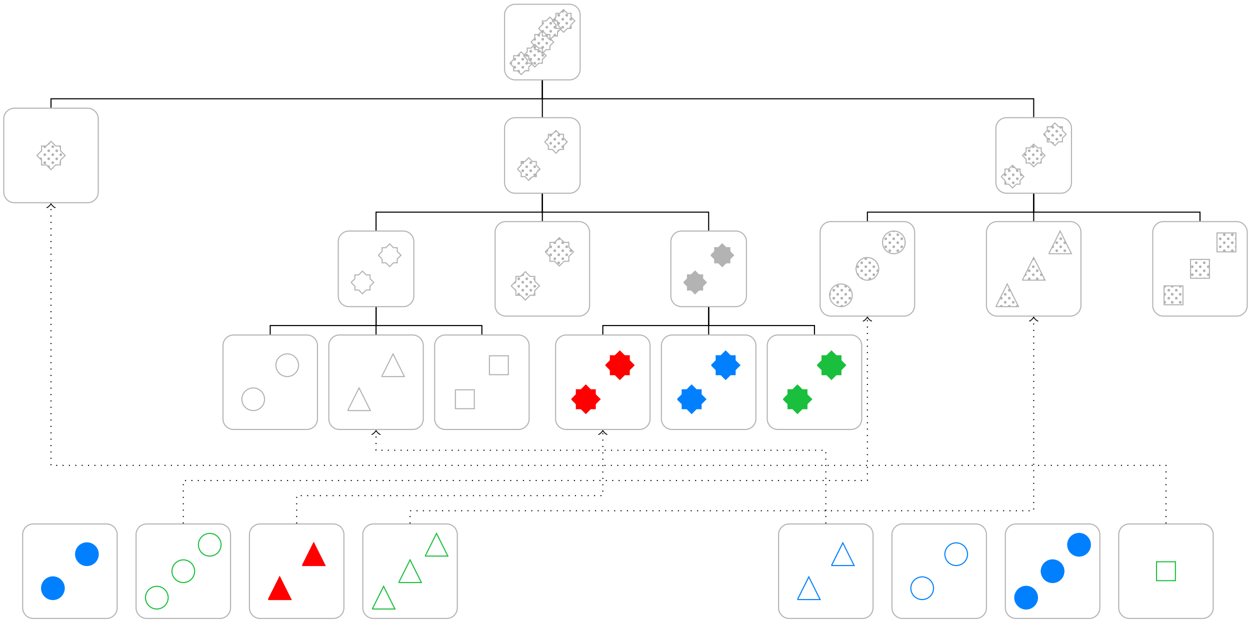
Third round: Player 1 decides to discriminate  on shape. Player 2 decides to discriminate  on filling. This leads to:



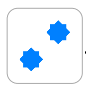
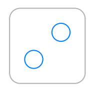
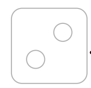
Fourth round: Player 1 assigns  to ; Player 2 assigns  to  This leads to:

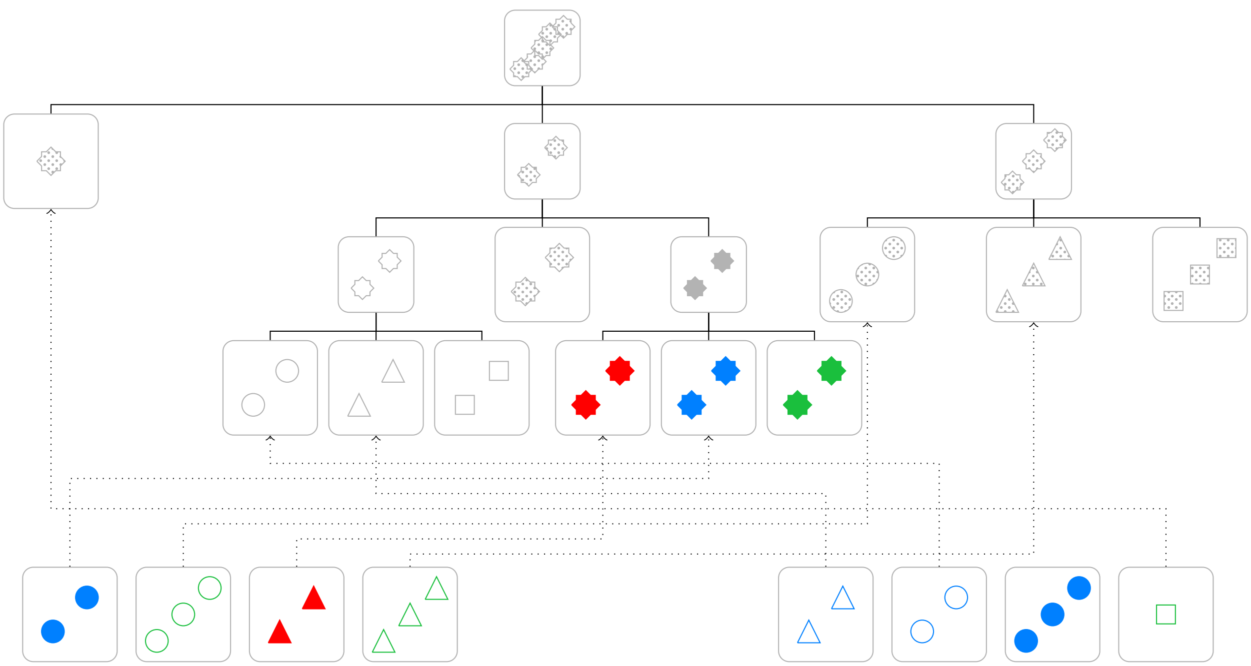


Fifth round: Player 1 discriminates  on color. Player 2 discriminates  on shape. This leads to:



Player 1 Player 2

Sixth round: Player 1 assigns  to . Player 2 assigns  to . This leads to:



Player 1 Player 2

Player 1 has won. This is the final classification.

7.9 Variations

– It is possible to play the same game with hands hidden from the other players;

– It is possible to play the same game with hands as decks from which players draw cards one by one, when they do not want to expand a leaf class.
– There could be a penalty (draw another card) when expanding.
– Or, on the contrary, a player may continue to play, as long as she creates subclasses.

7.10 Science

The game teaches how to create classifications that distinguish between all cards on the table and only these. This can allow later to identify the objects when they are encountered again.

In this game, the classification is partly determined by these cards: there is no need to make distinction that do not discriminate two classes. It is also determined by the way players play: from the same starting point, depending on their actions, there may be different classifications.

This shows that there is not one unique classification: several such classifications may be equally useful for discriminating objects. This is the way many machine learning algorithm work: learning to discriminate objects by using their different features.

This also shows that these classifications are not universal, they depend on the cards to discriminate and, if new cards are drawn, the classification may be extended.

7.11 Questions

1. * What is the maximum depth (number of successive forks) of a classification?
2. ** What is the smallest number of leaf classes in a classification of maximum height?
3. ** And what is the maximal number of leaf classes?
4. ** What is the number of classes in the largest classification?
5. *** How many largest classification are there?
6. *** How many classifications are there?
7. *** Is the resulting classification dependent from the card played and their order or will this produce a different classification?

7.12 Thoughts to be shared

– It is easy to build a classification from observation. By splitting classes with one separating characteristic, and starting from the top-class with no characteristic set, it is possible to create classifications in which each card belong to exactly one most specific class.
– There does not seem to be a single classification.
– Classifications do not need to be 'complete' to be used.

7.13 Knowing more/doing more

– It is time to play with the *Class?* game of Classifying in an unknown classifications.

Game 8

Classifying in an unknown classification

8.1 Audience

Ages 8 and up.

8.2 Duration

45 minutes–1 hour (depending on the amount of rounds played).

8.3 Resources

- Cards;
- Classification boards (either printed or generated on computer).

8.4 Learning outcomes

- Learn that without knowing the classification, it is possible to properly assign cards to classes.

8.5 Prerequisite

- Know what a class is (What is a class?).
- Know what a classification is (Subclasses).
- Identify the class in a classification to which a card belongs (Identifying with classifications).

8.6 Skills

- Logical reasoning.

8.7 Game play

This is the first part of the *Class?* game. It should now be easy to play.

1. Each player has ten cards, one player has a classification (that is kept secret and must be respected during the game).
2. One card from the deck is turned, this is the first class.
3. The player with the classification starts.
4. Players play turn by turn placing one or more cards from their hand on the table that must belong to the same leaf class:
   – either by adding them to a class already on the table,
   – or by creating a new class. The player with the classification can create a new class only if he cannot add to an existing class. When creating a new class, players must then draw a new card from the deck that they add to their hand.
5. At each turn, the player knowing the classification tells if this is correct or not with respect to the classification. If not, the player takes his cards back and takes one additional card from the deck.
6. The first player with no card wins.
7. At the end of the game, each player scores as many points as there are cards left. After several rounds (players may exchange roles), the player with the least points wins.

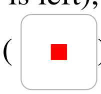
Note

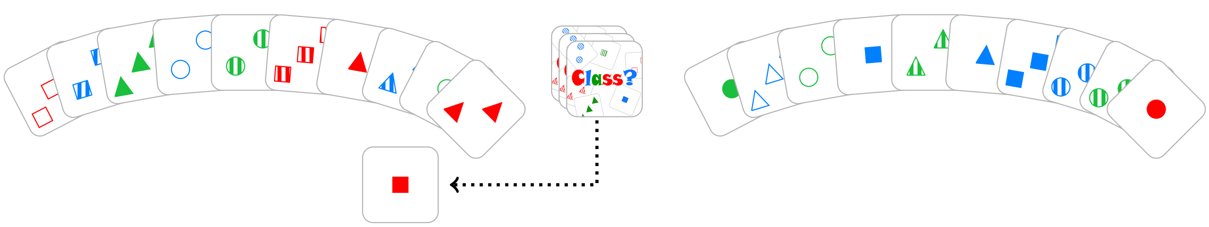
– All the cards that are played in one turn must be played to the same leaf class,
– Players are not required to play all their cards belonging to a leaf class *at the same time*.

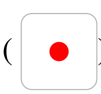
8.8 Example

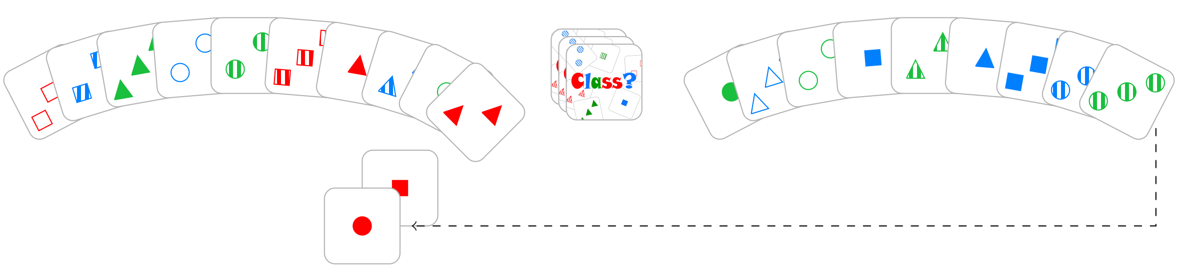
We take inspiration from the example given in the rules for the game.

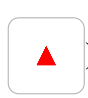
At the beginning (the guesser is left), the knower (right) has the classification of Section 7.10, p.25).

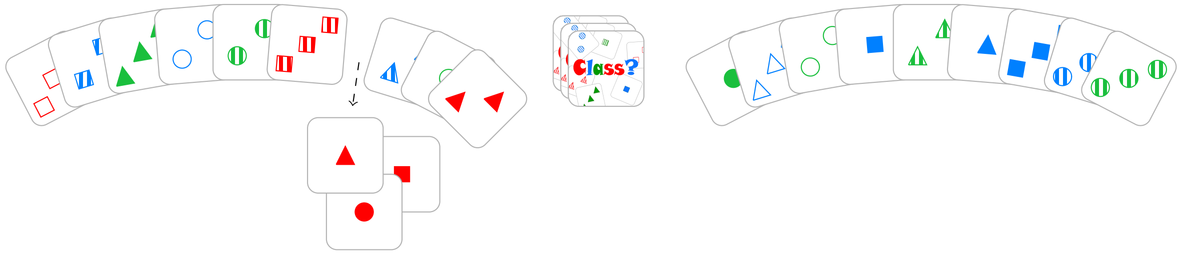
One card is drawn from the deck () to make the first class:

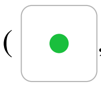
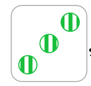
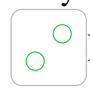
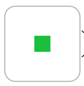


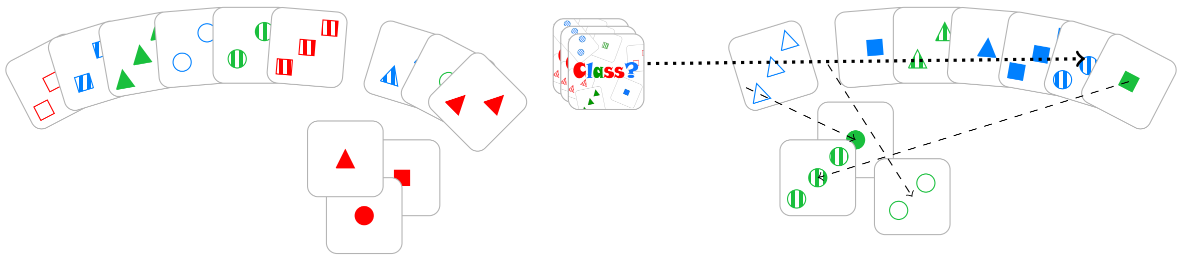
The knower (right) is obliged to add the unique card from this class that he has ():

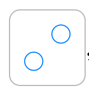
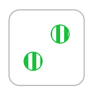
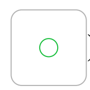


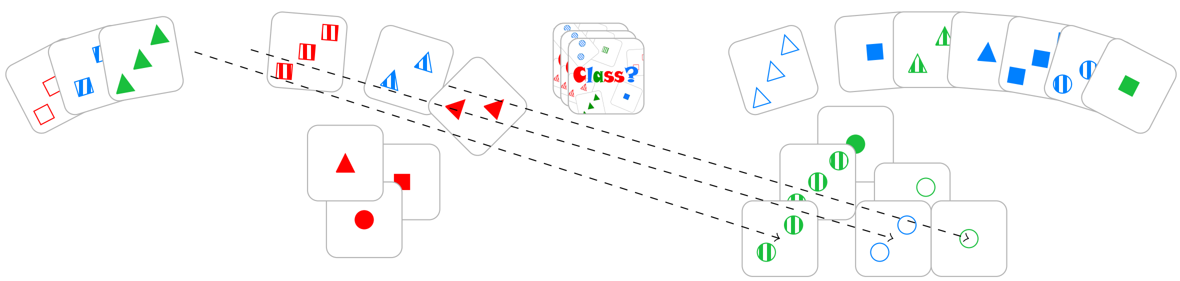
The guesser (left) adds one card () to the class, it is correct:



The knower (right) cannot complete the current class, which is already complete. So she creates another class with three cards featuring green circles (, , ) and draws one new card from the deck ():



The guesser (left) plays three cards (all its circles: ,  and ):



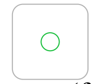
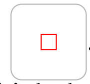
This is incorrect: he takes his cards back and draws a new card from the deck. And so on until one of the players has exhausted her cards.

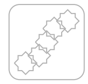
8.9 Variations

- The knower may announce the number of leaf classes at the beginning.
- The game may be played with multiple guessers.
- It is possible to divide the set of cards in half and play two games in parallel, since in general we do not exhaust all the cards.
- It is possible to play twice with the same classification but with two different guessers.

8.10 Science

There may be many different classifications over the same set of cards. However, it is possible through trial-and-error to find out if cards are classified together or not.

It is even possible to develop strategies in order to find this more surely. For instance, once establishing that some characteristic values are possible in a class (for instance because it contains  it can contain any of its characteristics values: one green empty circle) and that other are indifferent (for instance color and shape because it also contains ), then it is possible to attach .

This kind of reasoning can also be played by inducing the most specific class to which the two former cards belong: . It is then clear that the latter card belongs to this class as well. It does not mean that the class is described by , for instance, it still may be  or .

Similarly, if another class contain the card  then not all card containing one element are in the same class. It is also sure that  belongs to yet another class.

8.11 Questions

1. *** What happens if you delete a card category (for example all red cards)? And what happens if we add one?
2. *** Can the game work if instead of the question “What color is the card?” we ask ourselves “Is the card green?”. What would be the consequences?

8.12 Thoughts to be shared

- Everyone can organize their knowledge in the way that seems most useful for them, the most appropriate: there is no “good” representation.
- This does not prevent from learning very quickly how cards must be classified.

8.13 Knowing more/doing more

- Try Guessing the classification.

Game 9

Guessing classifications

This is the second part of *Class?*.

9.1 Audience

Ages 10 and up.

9.2 Duration

20 minutes.

9.3 Resources

- Cards;
- Classification boards (either printed or generated on computer).

9.4 Learning outcomes

- Learn that from a few examples it is possible to guess a hidden classification.

9.5 Prerequisite

- Know what a class is (What is a class?).
- Know what a classification is (Subclasses).
- Identify the class in a classification to which a card belongs (Identifying with classifications).

9.6 Skills

- Logical reasoning.

9.7 Game play

1. The table features several leaf classes as sets of cards. This may be the state of the table at the end of playing Classifying in an unknown classification.
2. Each player must describe the classification corresponding to these classes.
3. The player who describes it correctly wins.

When part of the *Class?* game, in the event of a tie, the player who guessed the classification correctly most often wins.

Note

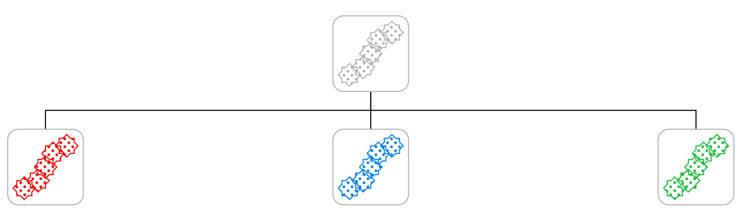
– The description of the classification may be given by explaining by which characteristic each class is split starting from the top class (see Example).

9.8 Example

Consider the state of the table at the end of the example of the previous game. It features two classes:

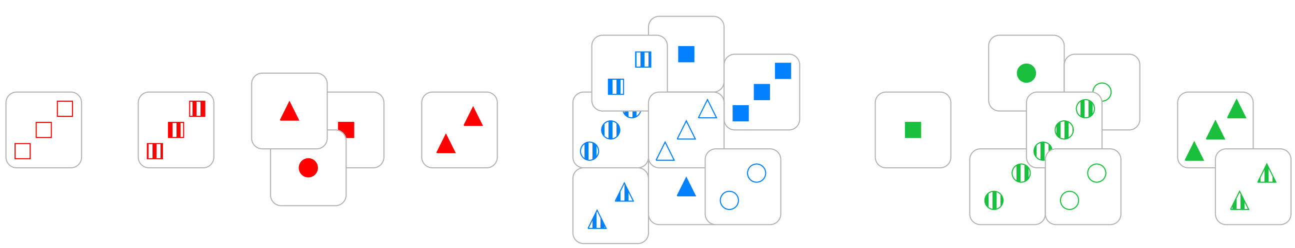
What can be stated about the actual classification (Section 7.10, p.25)?

By analysing, these two classes, it can be observed that the left-hand class has cards with the same number (1), same colour (red) and same filling (filled), and the right-hand side class has cards with the same colour (green) and the same shape (circle) but different filling and number. A classification starts from the top by dividing classes along one single characteristic. From these two classes it is possible to deduce that this characteristic is *not* shape, from the left-hand class, and *not* number nor fillings, from the right-hand class. Hence, it can be deduced that the first characteristic along which the top is divided is colour. It is thus this way:



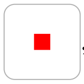
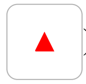
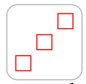
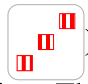
Actually, it is not possible to deduce more from these two classes.

For the sake of the exercise, consider that all cards that the players had in their hands, have been dispatched. This would lead to the following classes on the table:

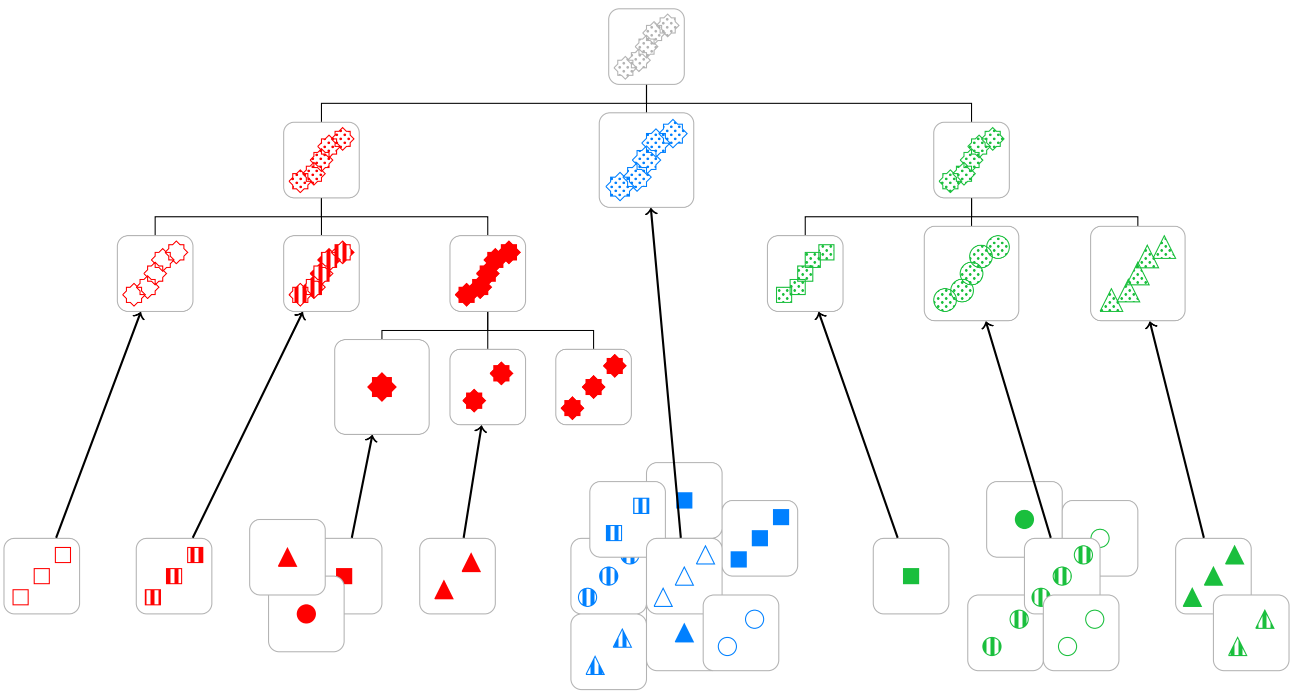


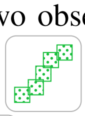
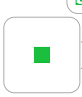
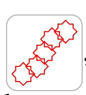
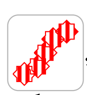
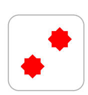
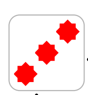
A first remark is that the cards in the blue class vary in number, shape and filling. Hence, the blue class is not further subdivided.

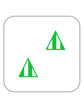
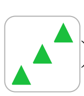
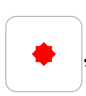
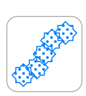
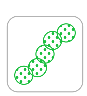
Then, on the green side, the three classes have homogeneous shapes and at least one of them differs from filling and numbers. Hence, the dividing characteristic can only be the shape.

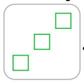
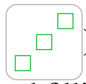
Finally, on the red side, there are four classes, hence subdivision. One of these classes contain heterogeneous shapes ( ,  and ), hence the shape does not divide the classes. Remains the number and the filling. However, two classes ( and ) differ *only* on the filling of their cards. Hence, the first dividing characteristic is necessary the filling. Then, there are two classes with same filling (full), hence the class  is necessarily subdivided. Since, one of these has heterogeneous shape, their only differing characteristic is number, thus it is divided by number.

This information leads to the following classification:



Two observations can be made here. On the one hand, the classification is not yet complete: the class  is further subdivided, but the players cannot know this because they have only one card (). This reflects uncertainty with respect to the status of this class on the side of the players. A similar uncertainty occurs for classes , ,  and . This is the reason why they have been kept smaller, although they are leaves at the moment: there is no way to know if they are further subdivided or not.

On the other hand, the players know that the class  is not futher subdivided because the cards belonging to it ( and ) do not share other characteristic values than those of the class. The same observation holds for classes ,  and  for similar reasons.

Having more cards on the table obviously helps to decide if the classes are further divided. But it is not always easy to decide. For instance, consider that there is an additional class based on a single card: . This would help players to find out that  is further subdivided because the two cards belonging to it ( and ) are assigned to different classes. However, because they differ on two characteristics, number and filling, it is not possible to know which one is the dividing characteristic. Hence, the players know that there are subclasses, but they also know that they do not know which ones.

9.9 Variations

– To play this game independently from Classifying in an unknown classifications, it is possible that one the knower draws a classification and 15 cards that she dispose on the table according to the classification. Then the guessers have to guess it.
– Another game would consists of drawing a classification and 16 cards at random, and arranging them on the table according to a classification. Then, each player, in turn, have the right to withdraw one card (alternatively, as many cards she wants) from one single class, so that it is still possible to guess the classification. The last player to withdraw a card wins.
– This game suggests another game, that can be played alone: take a classification, put on the table a minimal number of cards, arranged in classes, which correspond to only one classification. Is it always possible?

9.10 Science

By not knowing an exact classification, but only some groups of cards belonging to the same leaf class, it is sometimes possible to guess the actual classification. Sometimes, the groups of cards do not provide enough information to do it, or to do it with certainty, but often it is possible. Sometimes it is also possible to be sure of the unique classification leading to such leaf classes. In this case they provide a proof of this.

9.11 Questions

1. * Is it always possible to guess the classification?
2. * If we exhaust all the cards of the deck, do you think we will know exactly what the classification is?
3. ** Can this capability to guess the classification be useful to play Classifying in an unknown classification?
4. *** Could you design a procedure to build the minimal valid classification?

9.12 Thoughts to be shared

– It is possible to guess a classification from a few examples with yes/no answers.
– There are situations in which we do not guess the full classification, because some classes may not be played at all. Even though, we can still communicate. We do not need complete knowledge to communicate.
– Even if we use all the cards, we might not know what the classification is because we do not know in what way the classes are organized.

9.13 Knowing more/doing more

- Check the *Class?* rules.
- Check out Future developments (beyond *Class?*).
- The *Class?* game can be though of as a Eleusis or Eleusis express game with classifications.

Game 10

Future developments (beyond *Class?*)

Various more sophisticated games may be considered from here:

Teaching classifications The goal of a player is that the opponents learn the classification (without words of course). This should help understanding the importance of disclosing information or not.

Evolving classifications Players may modify their classifications to better communicate with others... Does this lead to the same classification? This game could be very related to https://classic.csunplugged.org/wp-content/uploads/2014/12/PhylogeneticsUnplugged.pdf.

Ordering classifications It is possible to order classifications, because some are more precise than the others. It would also be possible to take advantage of identification in one classification, to identify in another. This could be the basis of yet another game.

Matching classifications Both players have a classification and should build an alignment between their classification through guessing the position of cards in each other classifications.

It is also possible to render the games more interactive. For instance, it is possible to introduce the possibility to ask questions to the oracle. These could be questions like: 'does this card belong to this class?' Should it be one question by turn? By game? Should these cost points?

It may also be possible to introduce gossip by allowing players to communicate information, true or false, in order to help each others.

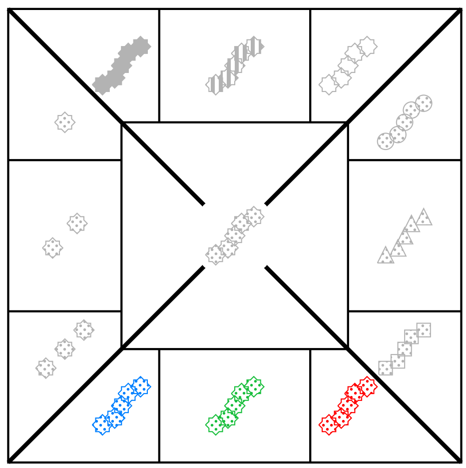
For each of these games, it is possible to ask for an algorithm that plays the game. Algorithms may embed explicitly the strategies developed by players.

On a more 'plugged' side, we have developed a computer program to play *Class?*. It features automatic players with different strategies. It also provides an application programming interface (API) so that anyone can implement their own strategies and see if they work well. This may be the occasion for 'hackathons', contests and experimenting with learning strategies. This may be taken as an opportunity to introduce different techniques such as constraint programming or general game playing in order to program the game.

Appendix A

Class template

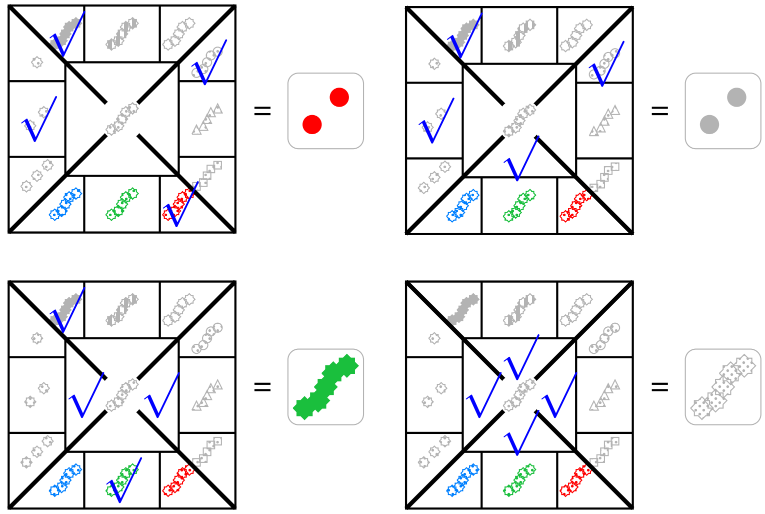
In order to avoid cheating, it is possible to write down the chosen class, or mark it with the following template:



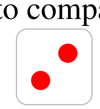
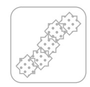
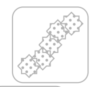
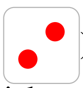
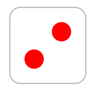
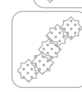
It describes the class characteristics in a square divided in four along the diagonals:
- top: filling;
- bottom: colour;
- left: number;
- right: shape.

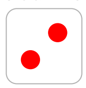
Each quarter is divided in four parts: three parts along the border corresponding to the possible value of the characteristic, and one part in the middle corresponding to the absence of value of the characteristic.

Any class can be described by selecting exactly one part in each quarter (note that this will correspond to $4^4 = 256$ possible classes, as expected). Hence:

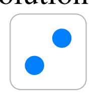
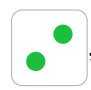


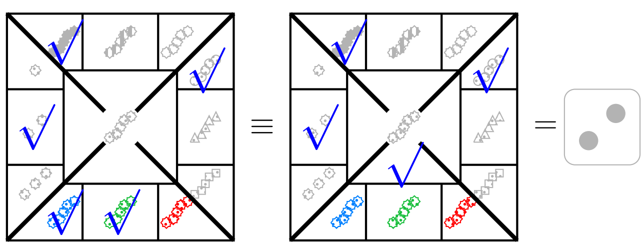
Do you see how it is possible to compare two such classes?

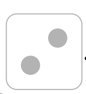
It is possible to see that class  is subsumed by class  (or class  subsumes class ) because one can go from the template of class  to that of class , by only pushing ticks outwards (or inwards for subsumes).

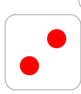
This is not possible between  and  because none subsumes the other.

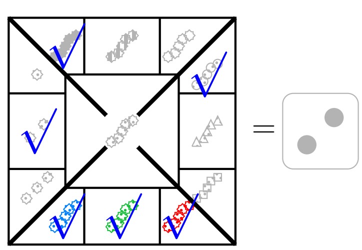
It is also possible to use such templates as an aid for tracking solutions.

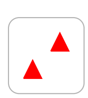
For instance, in the case of Game 1, after being presented with  and , the template could be ticked as:

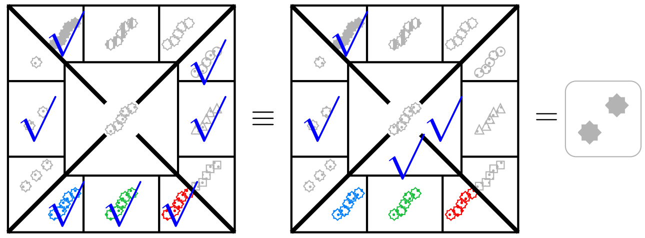


which indeed corresponds to .

Then, when adding  this provides



corresponding to the same class, but when adding  the result becomes



The template can be found from the *Class?* web site as all other material.

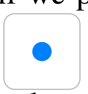
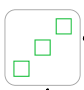
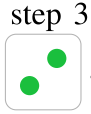
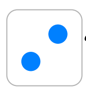
Appendix B

Answers to questions

Here are the (very) short answers to the questions. They are here for the reader to confirm answers.

If you do not agree with some answer or think that you have better questions, think about it and if you are convinced, contact us. Detailed answers to questions may be available elsewhere.

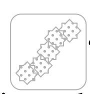
B.1 What is a class?

1. * Does each belong to a unique class? No.
2. ** Does each belong to a unique smallest class? Yes.
3. ** How many different classes are there? 256.
4. ** What if we pick cards that do not share any characteristic (color, shape, texture or number), for instance  and ? Is there a class to which they both belong? Yes.
5. *** Is the class chosen in step 3 of the game dependent on the order of picking cards? Is there a difference if we first pick  and then ? No.

B.2 Card splitting classes

1. * How many combinations of sets of three cards are there? 120
2. ** Is it always possible to find such a class? No.
3. ** If it is possible, is the class necessary unique? No.

B.3 Guessing classes

1. ** How many cards are necessary to determine the class of all cards, ? 2.
2. ** Do the examples and counter-examples always correspond to a unique class? No.
3. *** How many cards do we need to play to be sure to have determined the class? It depends. The minimum is 2, the maximum 46.

B.4 Subclasses

1. * Are each pair of classes in subclass relations? No.

2. * Is there a class that is in subclass relation with all other classes? Yes.
3. ** What other relations between classes can you think of? Give examples. Disjoint, when the classes do not have any cards in common, or Overlapping, when they have some classes in common but none subsumes the other.

B.5 Disjointness

1. * Are classes containing one single card necessarily disjoint (have no card in common)? Yes.
2. ** Could both players have the same positive class in the end? Yes.

B.6 Identifying with classifications

1. * What is the maximum number of steps to make to reach a leave? 4.
2. * Can there be two cards classified in a leaf class? Yes.
3. ** Does a card always belong to the same class in all classifications? No.

B.7 Building classifications

1. * What is the maximum depth (number of successive forks) of a classification? 4.
2. ** What is the smallest number of leaf classes in a classification of maximum height? 9.
3. ** And what is the maximal number of leaf classes? 81.
4. ** What is the number of classes in the largest classification? 121.
5. *** How many largest classification are there? 55296.
6. *** How many classifications are there? 12810097696001.
7. *** Is the resulting classification dependent from the card played and their order or will this produce a different classification? Yes. Just try it! Do it several times but with the same set of cards.

B.8 Classifying in an unknown classification

1. *** What happens if you delete a card category (for example all red cards)? And what happens if we add one? The game has the same difficulty, just less or more cards.
2. *** Can the game work if instead of the question "What color is the card?" we ask ourselves "Is the card green?". What would be the consequences? Yes this would work, but the classification would be binary (with two child for each node: green/non green, square/non square, etc.) and their height would be higher.

B.9 Guessing classifications

1. * Is it always possible to guess the classification? No.
2. * If we exhaust all the cards of the deck, do you think we will know exactly what the classification is? No.
3. ** Can this capability to guess the classification be useful to play Classifying in an unknown classification? Yes.
4. *** Could you design a procedure to build the minimal valid classification? Sure you can, try it!